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Suong V. Hoa

4D PRINTING OF COMPOSITES



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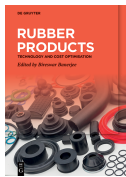


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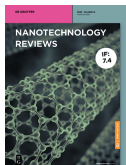
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Suong V. Hoa

4D Printing of Composites

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Preface

Work on long continuous fiber polymer matrix composites has been ongoing for more than 70 years. There are more and more applications of these materials in making many engineering structures such as aircraft, automobiles, wind turbines, and sports equipment. The manufacturing of these structures has been mostly by hand. Recent advances in automated manufacturing technologies have introduced the use of automated tape laying (ATL) and automated fiber placement (AFP). The manufacturing of structures using composite materials is basically an additive manufacturing process, where layers of the materials are deposited over previous layers. This is the case whether the manufacturing is done by hand (hand lay-up) or by machine (ATL or AFP). Recently, the surge of 3D printing has brought more focus on the method of additive manufacturing. However, 3D printing normally refers to the use of powder, paste, or liquid forms of materials such as plastics or metals. For products made by 3D printing using plastics, the materials usually do not have high strength and modulus. There have been attempts to incorporate long continuous fibers into the process using regular 3D printing machines. However, the amount of fibers that can be incorporated is usually low (less than 10% by volume). On the other hand, ATL or AFP machines handle composite preregs, where the fiber volume fraction is on the order of 60%. It is along the line of these ATL and AFP machines that the concept of 4D printing of composites (4DPC) was formed.

Over the past few years, there has also been rapid development of materials for 4D printing. This process is an extension of 3D printing. In this process, materials of different properties are deposited onto a flat mold (or molds of simple geometry). During the material deposition process, different materials are deposited at different locations onto the flat mold. These flat layers (or stacks of layers) of the material are then subjected to some external stimuli. The different materials react differently to external stimuli such as the absorption of water (or other liquids), heat, or light. The configuration of the spatial variation in material properties needs to be designed such that upon excitation from the external stimuli, the flat material structure would curl up to make some curved structure. Using this technique, structures of complex geometries can be made without the use of molds with complex geometry. This method usually uses materials that have low modulus and strength.

The concept of 4DPCs extends the concept of 4D printing but uses long continuous fiber composites that have been used for many years in industries. As such, stiff and strong structures with complex geometries can be made using molds of simple geometries such as flat plates. These structures can be classified into three categories: U-structures, C-structures, and A-structures. U-structures are unconstrained structures. These are the curved pieces resulting from the shape transformation from the flat stack of layers. These structures have free edges. If the edges are constrained by bonding, they become C-structures. While the U-structures may change shape due to variations in ambient conditions such as temperature and moisture, C-structures do not.

One particular type of C-structure is the A-structure (“A” stands for almost). This refers to structures like the A-cylinder or the A-cone. They have the same shape as the cylinder or cone and may have mechanical properties that are similar to the perfect cylinder or cone, except that the edges of the A-cylinder or A-cone are bonded.

There are many applications of structures made by 4DPC. One application for a U-structure can be in the area of effective packaging. In this case, flat shapes are shipped to remote locations. Upon arrival, thermal activation can be used to change the flat structure into a curved structure. This is particularly useful for sending shelters to refugee camps or for sending supplies to outer space. Another application for the U-structures can be a cover for space instrumentation. During the evening, when the sun is not shining, the composite structure closes to protect the instrumentation. During the day, it opens up to expose the instrumentation to sunlight. One application for the C-structures is the corrugated core for a flexible wing. The corrugated composite core is made by 4DPC. Subsequently, it is bonded to the upper skin and lower skin to enable the manufacturing of a flexible wing. One application for the A-structures is the A-cone. The 4DPC process allows the composite cone to be made without the need for a mold of conical shape. It was demonstrated that this A-cone has the same buckling strength as a cone of similar weight made using the process of filament winding.

This book presents the development of 4DPCs since the inception of the concept in November 2016.

Montreal, June 2024

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I thank many students, assistants, and associates who have made contributions to the work on 4D printing of composites. I thank Dr. Tsz Ho Kwok for giving a seminar in the summer of 2016 that brought to my attention to the term “4D printing.” I thank Dr. Daniel Rosca for having made many early structures. I thank many students, including Xiao Cai, Manpreet Singh, Bhargavi Reddy, Victor Raju, Marjan Abdali, Jasmin Anick, Mahmoud Fereidouni, Emad Fakhimi, and Dr. Hamid Hamidpour, who have worked on various aspects of the technology. I thank Heng Wang for having done most of the drawings.

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Part I: **Fundamentals**

Chapter 1

Introduction to composite materials, laminate theory, and extended laminate theory

1.1 General

Composites are materials that consist of at least two phases such as a hard, strong phase and a soft, weaker phase, with a strong bond between them. The hard phase provides strength and stiffness for the composite material, while the function of the soft phase is to hold many individual entities of the hard phase together. One commonly used type of composite material is fiber-reinforced polymer materials, and is the focus of this book. In these materials, the hard phase has the configuration of long, slender fibers with small diameters, and the soft phase is usually some type of polymeric materials. The commonly used fibers are carbon fibers, glass fibers, and Kevlar fibers. The diameters of these fibers are between 7 and 10 μm . The commonly used soft phases (also called the matrix material) are thermosets such as epoxy and polyester, or thermoplastics such as polypropylene, polyethylene, polyphenylene sulfide, or polyetheretherketone. For the 4D printing of composites (4DPC) considered in this book, only thermoset resins such as epoxies are considered. More information on manufacturing using other types of matrix materials can be found in reference [1].

1.2 Characteristics of fiber-reinforced epoxy composite materials (from manufacturing considerations)

The manufacturing of composites made of fiber-reinforced epoxy materials depends on the curing behavior of the epoxy resin. The epoxy resin is a thermoset resin system. This means that the resin system consists of two types of molecules: the main molecules and the linking molecules. Figure 1.1 shows schematic representation of these molecules and their interactions. Figure 1.1a shows an ensemble of the main molecules. For an epoxy system, these molecules can be diglycidyl ether of bisphenol A, epoxy novolac, and so on with different combinations of molecular weight and/or types of these molecules. These molecules have short molecular lengths. As such, they have low viscosity (normally in the range from 50 centipoise (cP) to several thousand cP at room temperature). As such, these resins can flow like a liquid, and this facilitates the wetting of the fibers during the manufacturing process.

The low viscosity of the resin is an advantage for the wetting of the fibers for manufacturing. However, after the fibers are wetted, it is necessary for the resin to harden in order to provide strength and stiffness to the final composite material. One way to do this is to link these molecules together to make a network. This is done by

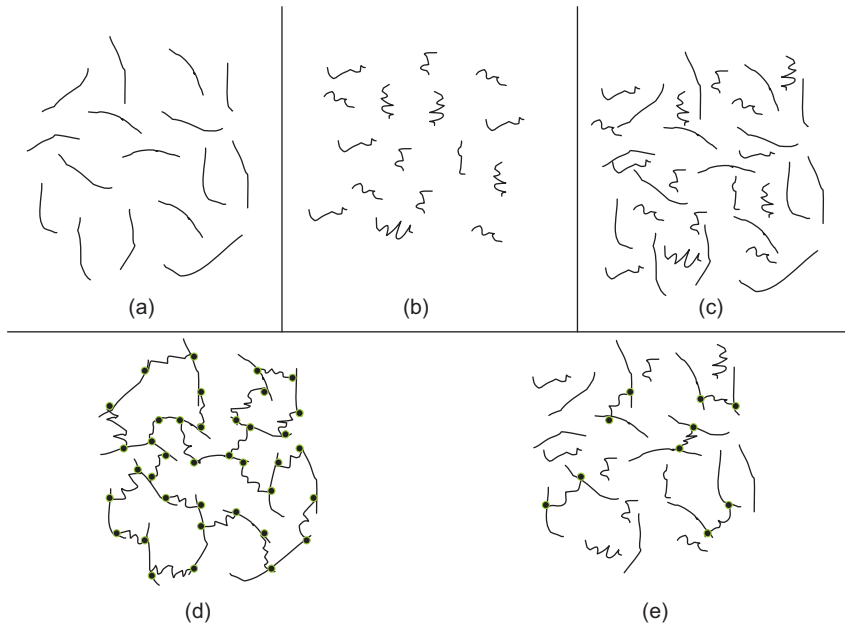


Figure 1.1: Schematic representation of the molecules in a thermoset resin system [1].

using the second type of molecules, called the linking molecules (Figure 1.1b). For epoxy resin systems, the linking molecules can be diethylenetriamine or similar types of amine or anhydride curing systems. Normally, the linking molecules are mixed together with the main resin molecules during the manufacturing of the resin system (Figure 1.1c). The linking molecules link the main molecules together through chemical reactions. The linking action is usually called the curing process. This chemical reaction usually requires the activation of heat or the presence of some type of catalyst. Since there are millions and millions of these molecules, the linking reactions take time. After all linking reactions are complete (all links have been made), the curing is complete, and a rigid network is obtained (Figure 1.1d). The degree of cure of the resin system is usually denoted by the symbol α , and it varies from 0 to 1. At the beginning, when there is no linking reaction at all, the degree of cure is 0, and the material is a liquid. After all links have been made, the degree of cure is equal to 1, and the material is hard and stiff. At that time, the hard resin together with the strong fibers provides the composite materials with the strength and stiffness desired. Figure 1.1e shows the partial state of cure of the resin where only a small fraction of the cross-links have been made. This results in a flexible solid state of the resin. When fibers are embedded in this resin state, the product is called a prepreg (pre-impregnated).

1.2.1 Transformation behavior of thermoset resin during curing

Long, continuous, fiber-reinforced polymer composite materials consist of two main components. One is the long, continuous fibers, and the other is the resin system. During the curing and cooling process, the dimensions of the long fibers do not change much. Most of the deformation of the composite materials is caused by the deformation of the resin matrix. To simplify the presentation, the deformation of the epoxy matrix resin is presented here, even though the behavior of other thermoset polymer systems can follow a similar process.

In the raw state at room temperature, most epoxy resin systems appear in liquid form. The liquid mobile monomer molecules can be considered as chemical entities that are discrete with respect to one another. They are said to each occupy a unit volume that is dictated by their van der Waals volume and thermal energy [2]. The curing of the epoxy resin is an exothermic process. During this process, the monomer molecules first undergo formation and linear growth through branching polymerization. Then the resulting molecular chains form cross-links so that a large rigid three-dimensional molecular network is formed.

The curing process is commonly described via three distinct regions (Figure 1.2). In region I, the resin is uncured and behaves as a viscous fluid. Region II denotes the curing stage of the resin. During the curing stage, there is a significant reduction in volume due to the reduction in the van der Waals space between the molecules. When the degree of cure is sufficiently large (not necessarily 100%), the resin reaches a gel state. One way to describe the gel is that it is no longer a liquid. As the degree of cure increases to close to 100% (region III), the resin reaches the vitrified state. One way to describe vitrification is that it is a rigid, solid state. The state between gel and

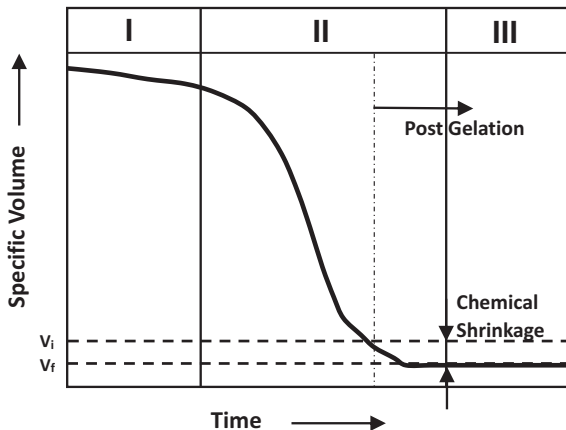


Figure 1.2: Three distinct regions in which the physical properties of the resin change throughout the curing process.

vitrified therefore can be described as a viscous material, and not as a liquid or a rigid solid. Once the material has reached the vitrified state and if the temperature does not change, its volume does not change.

The change in volume of the resin material may create distortion in the composite structures. In the case of 4DPCs, the distortion due to resin shrinkage may contribute only a small portion of the total change in shape. This is because during the first phase of resin shrinkage, the modulus of the resin is not sufficiently large, and the shrinkage in the resin does not provide sufficient force to get the fibers to move along due to the stiffness of the fibers. The distortion only occurs when the resin modulus becomes large enough. The implication of this is that not all volume reduction in the resin will translate into distortion in the composite structures. For an epoxy system consisting of diglycidyl ether of bisphenol F and diethylene toluene diamine, information from reference [3] can be fitted with the following equation:

$$VS = 0.0645\alpha + 0.0073 \quad (1.1)$$

where VS represents volumetric shrinkage and α represents the degree of cure.

From this equation, the total volumetric shrinkage of this resin system is about 6%.

The same reference [3] also shows the Young's modulus as a function of the degree of cure and temperature. The data can be fitted with the following equation:

$$E_m = 2.282 \alpha + 1.113 \quad (1.2)$$

where E_m is the resin modulus in MPa.

Note that eq. (1.2) is applicable for a degree of cure greater than 0.65. This is because below a degree of cure of 0.65, the resin may not be solid enough to exhibit a solid modulus. The range of the resin modulus is about 2.6–3.1 MPa. For the degree of cure ranging from 60% to 90%, the change in the volumetric strain is only about 1%, and not the whole amount of 6%. As such, the amount of volumetric reduction that may contribute to the deformation of the laminate during curing is only about 1%, and this corresponds to a resin modulus between 2.6 and 3.1 GPa.

The solid resin modulus varies with temperature. The variation of the resin modulus over the range of temperatures from 300 °K (27 °C) to 450 °K (177 °C) can be fitted with eq. (1.3) from the data in reference [3]:

$$E_m = 0.0072 T + 5.704 \quad (1.3)$$

where the resin modulus is in MPa and the temperature T is in kelvin. Note that over the temperature range from 27 to 177 °C, the modulus varies from 3.52 to 2.40 MPa.

Assuming that the resin is already completely cured before it is cooled from the cure temperature to room temperature, there is no more resin shrinkage due to chemical reaction. Rather, during this cooling down, the deformation of the laminate is due completely to the difference in the coefficient of thermal contraction among the different layers of the laminate.

1.2.2 Coefficients of thermal expansion (or contraction)

Kravchenko and coworkers [3] investigated the chemical and thermal shrinkage in thermosetting prepregs for an out-of-autoclave toughened resin carbon fiber composite, CYCOM 5320-1, manufactured by CYTEC Engineered Materials. They found that the coefficient of thermal expansion (contraction) of the resin below T_g as follows:

$$\alpha_m = (-133.56a + 174.27) \times 10^{-6} \quad (1.4)$$

where a is the degree of cure.

In the case of a complete cure ($a = 1$), the value of the expansion coefficient is

$$\alpha_m = 40.71 \times 10^{-6} \quad (1.5)$$

Note that the coefficient of thermal expansion (contraction) depends on the type of resin system.

1.2.3 Prepregs

For handle-ability of the material during the manufacturing process, an intermediate configuration of the materials system called prepreg (pre-impregnated) is usually used. In this configuration, the degree of cure of the resin is about 0.3. This means that the resin is partially cured (Figure 1.1e) with embedded fibers. As such, it is neither a liquid nor a solid. Rather, it is a viscous material that does not flow, and it is soft. This prepreg is made by either allowing for some limited heating time to allow curing, and then the temperature is reduced to retard curing, or by manipulation using accelerators and inhibitors. This configuration allows handle-ability of the material. Prepregs usually appear in the form of sheets. The thickness of a sheet of prepregs varies from about 0.125 mm to about 0.150 mm. They are similar to wet wall papers except that they contain stiff fibers and are sticky on both sides. Figure 1.3 shows a roll of the prepregs. The pliability of the prepregs allows them to be drapable such that fabricators can drape them over mold surfaces to make structures with complex geometries.

Prepregs are usually stored at low temperature conditions (around 0 °C to -5 °C) in order to retard the curing reaction. Whenever a manufacturer is ready to make the structure, prepregs are taken out of the freezer and allowed to thaw out. The prepregs are then draped over the surfaces of the mold to make the composite structures.



Figure 1.3: A roll of slit carbon/epoxy prepregs.

1.3 Lay-up sequences

As mentioned above, prepregs are thin sheets of material. The thickness of each prepreg sheet varies from about 0.125 mm to about 0.150 mm, which is as thin as a sheet of paper. They are also sticky on both sides. These sheets are placed on top of each other to build up the thickness to the desired dimension of the structure. For example, for laminates in wing skins of aircraft, the thickness is about 3 mm. This would require about 24 layers to be placed on top of each other.

The fibers within the prepregs are usually unidirectional, as shown in Figure 1.4a. The fibers within a sheet of prepregs are aligned along one direction. This makes the sheet of prepregs to exhibit anisotropic behavior, meaning the properties are different along different directions. The prepreg is stronger and stiffer along the fiber direction (called the L direction) and weaker in the direction transverse to the fiber direction (called the T direction). In order to compensate for the lack of strength and stiffness along the T direction, sheets of prepregs are stacked along different directions. For example, let the direction along the length of a structure be called x . Then the first layer of the prepreg can be such that its L direction coincides with x , the second layer can be laid such that its L direction is perpendicular to x , the third layer can be laid such that its L direction makes an angle of 45° with respect to x , while the fourth layer has its L direction making an angle of -45° with x . The angle θ between the L direction of a particular layer and the x direction of the structure is called the fiber angle of that layer (Figure 1.4b). The ensemble of all fiber angles for a particular laminate is called the stacking sequence (Figure 1.5).

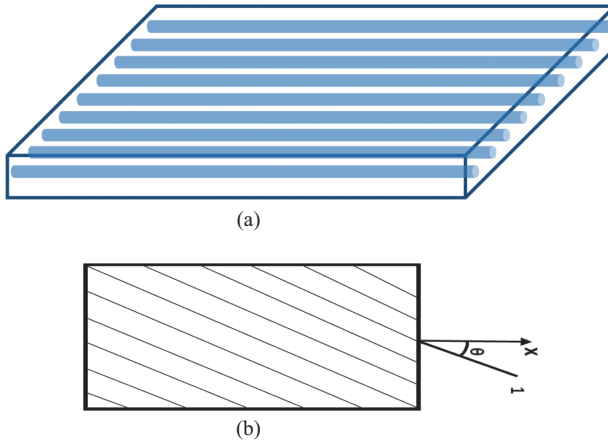


Figure 1.4: Unidirectional prepregs.

Figure 1.5 shows an example of the stacking sequence $[0/45/-45/90]_s$, where the subscript “s” means symmetric. Symmetric means that the fiber orientations have mirror symmetry about the mid-thickness. In other words, the above stacking sequence can be written in full as $[0/45/-45/90/90/-45/45/0]_T$, where the subscript “T” means Total.

The properties of the laminate (a stack of layers) depend on the material of the layers, the number of layers, the fiber orientation in each layer relative to the axes of the structures, and the stacking sequence. In terms of the stacking sequence, a laminate made of stacking sequence $[0/45/-45/90]_s$ would have some mechanical properties that are different from those of a laminate made up of the stacking sequence $[0/45/-45/90/0/45/-45/90]$, even if the materials and the number of layers are the same. Laminates made of lay-up sequence $[0/45/-45/90]_s$ are symmetric across the laminate mid-thickness while laminates made of lay-up sequence $[0/45/-45/90/0/45/-45/90]$ are not symmetric. Symmetric laminates do not exhibit significant amounts of distortion due to manufacturing, while unsymmetric laminates do exhibit dimensional changes due to manufacturing. Over more than 70 years in the development and applications of composite materials and structures, most composite structures are usually made of symmetric configuration. The reason for this is that symmetric laminates exhibit simpler behavior, making them easier to handle. Most, if not all, textbooks on composites have been dedicated to the behavior of symmetric laminates. A few examples are references [4, 5].

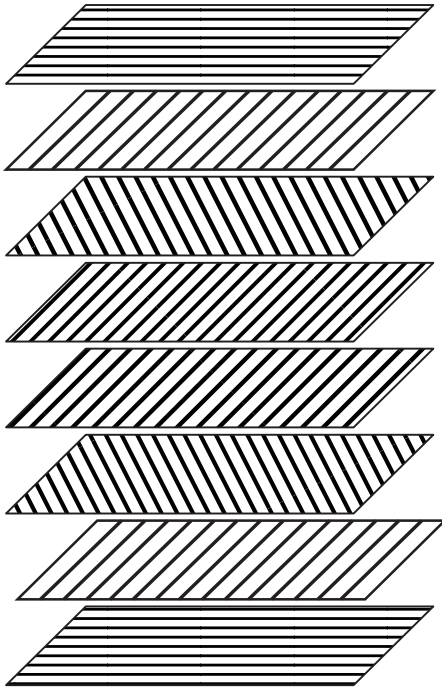


Figure 1.5: Symmetric stacking sequence $[0/45/-45/90]_s$.

1.4 Composite behavior during curing and cooling

After the prepregs are taken out of the freezer and allowed to thaw, the manufacturing of composite materials such as carbon/epoxy consists of the following stages:

- The prepregs are laid onto the surface of a mold.
- The prepregs are well-compacted using various means such as squeezing and debulking.
- Bagging materials are placed over the prepregs to provide a means for applying vacuum and pressure.
- The whole material assembly is placed either inside an autoclave or an oven for curing.
- Curing means heating to first reduce the viscosity of the resin. As the degree of cure increases, the viscosity increases. The cure temperature of most high-performance resin systems is about 177 °C.
- After the curing is complete, the material assembly is cooled from the curing temperature to the room temperature.

To understand the mechanisms responsible for the 4DPCs, the characteristics of the materials and laminate during curing and cooling are most relevant.

1.4.1 Material behavior during curing

During the manufacturing of thermoset composites such as carbon/epoxy, the material is subjected to a curing cycle, which is a schedule of heating and cooling as shown in Figure 1.6. The first phase of the heating is to reduce the viscosity of the resin. The reduction of the viscosity of the resin allows the resin to flow and wet the fibers thoroughly. The second phase of the heating is to increase the degree of cure of the resin, and this in turn increases the viscosity of the resin.

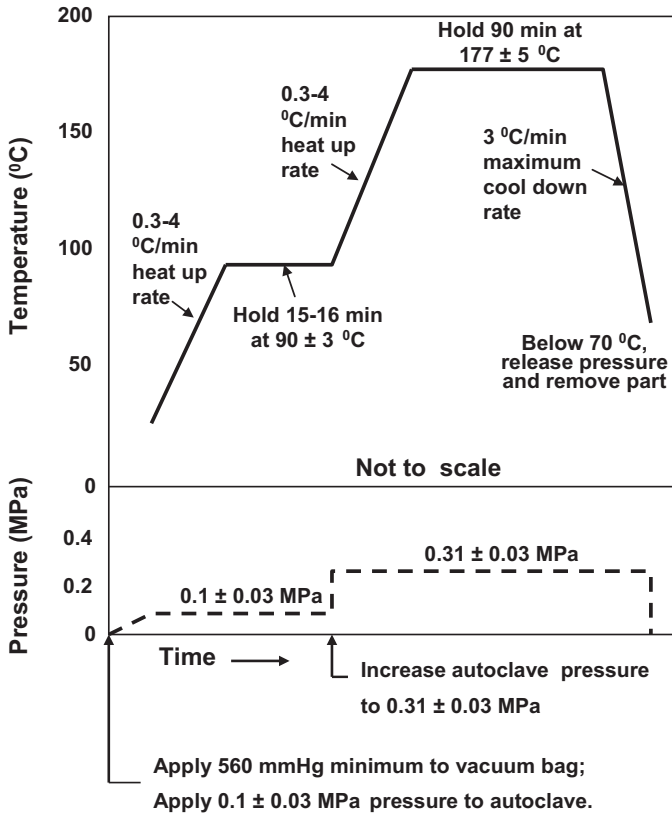


Figure 1.6: A typical curing cycle for thermoset composites such as carbon/epoxy.

The change in the viscosity of a typical resin system is shown in Figure 1.7.

During the curing of the resin system, chemical bonds are formed, and there is a certain degree of shrinkage in the volume of the resin material. At the same time, as the viscosity increases, the modulus of the resin also increases. The combination of the shrinkage of the resin together with the increase in material modulus can give rise to residual stress and deformation. The total shrinkage of the resin usually refers

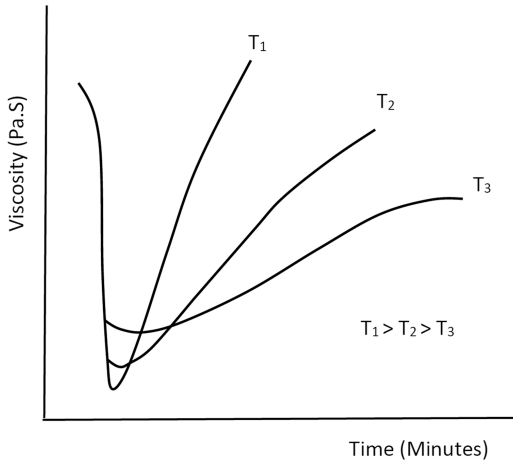


Figure 1.7: Change in isothermal viscosity of a resin system.

to the difference in the volume in the liquid state to the volume in the solid state. However, during the first stage of shrinkage, when the material is closer to the liquid state, the material modulus is low, and the combined effect of the shrinkage and the modulus is not sufficient to create a significant amount of residual stress and deformation. Only during the latter stage of shrinkage when the modulus is sufficiently large, the residual stresses and deformations are significant. As such, to determine the residual stresses and deformations, only a portion of the latter stage of shrinkage needs to be taken into account.

The concept of 4DPCs relies on the anisotropy of the composite structure, i.e., the laminates need to be unsymmetric. In conventional practice in composites, unsymmetry has always been considered a liability. For 4DPCs, unsymmetry is considered an asset. The unsymmetry of the composite laminate causes the laminate to exhibit deformation upon cooling from the cure temperature.

In the study of the stress and deformation of thin laminates, laminate theory has been used extensively to predict the behavior of symmetric laminates [4, 5]. It is simple and straightforward. Unsymmetric laminates may exhibit different shapes depending on their dimensions (ratio between thickness and length, and aspect ratio) and their temperature, as shown in Section 1.6 .

Even though most laminates analyzed using classical laminate theory have symmetric lay-up sequences, it will be shown later in this chapter that the radius of curvature of thin unsymmetric laminates, where the ratio between the thickness and side length (t/a) is small, approaches the radius of curvature calculated using classical laminate theory. Since classical laminate theory is simple compared to extended laminate theory (presented later in this chapter) and/or the finite element method, it is useful to present the analysis of unsymmetric laminates using classical laminate theory

here. The radius of curvature of unsymmetric laminates calculated using classical laminate theory can then be used to study the effect of different stacking sequences within the family of cross-ply laminates, or the effect due to thickness and material properties.

1.5 Laminate theory

1.5.1 General formulation

The analysis of thin laminates in many engineering applications has been successfully done using classical laminate theory. This is a linear theory and is based on the following major assumptions:

- The displacements are continuous throughout the laminate.
- The Kirchhoff hypothesis regarding the undeformed normal is assumed to be valid.
- The strain–displacement relation is linear.
- The material is linear elastic.
- The through-thickness stresses are small compared to the in-plane stresses.

The theory smears the individual lamina properties by integrating the constitutive equations through the thickness of the laminate. As a result of this integration, force and moment resultants are defined. Also, the A , B , and D matrices are defined.

As a result of the above assumptions, the strains at any point within the laminate can be expressed as follows:

$$\varepsilon_x = \varepsilon_x^o + zk_x \quad (1.6)$$

$$\varepsilon_y = \varepsilon_y^o + zk_y \quad (1.7)$$

$$\gamma_{xy} = \gamma_{xy}^o + zk_{xy} \quad (1.8)$$

where strain terms with superscript “o” represent the values at the mid-plane, given as

$$\varepsilon_x^o = \frac{\delta u^o}{\delta x} \quad (1.9)$$

$$\varepsilon_y^o = \frac{\delta v^o}{\delta y} \quad (1.10)$$

$$\gamma_{xy}^o = \frac{\delta u^o}{\delta y} + \frac{\delta v^o}{\delta x} \quad (1.11)$$

where u^o and v^o are in-plane displacements of points on the mid-plane.

The curvatures are assumed to be constant in the laminate and are defined as

$$k_x^o = - \frac{\delta^2 w^o}{\delta x^2} \quad (1.12)$$

$$k_y^o = - \frac{\delta^2 w^o}{\delta y^2} \quad (1.13)$$

$$k_{xy}^o = -2 \frac{\delta^2 w^o}{\delta x \delta y} \quad (1.14)$$

where w^o is the out-of-plane displacement of points in the mid-plane.

The strains in eqs. (1.6)–(1.8) are total strains. The mechanical strains (to be used in the stress–strain relations) are obtained by subtracting the thermal strains from the total strains. When that is done, the stress–strain relation at any point is given as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o + z k_x^o - \alpha_x \Delta T \\ \varepsilon_y^o + z k_y^o - \alpha_y \Delta T \\ \gamma_{xy}^o + z k_{xy}^o - \alpha_{xy} \Delta T \end{bmatrix} \quad (1.15)$$

where x and y are the coordinates parallel to the edges of the laminate. The $\overline{Q}_{ij}$ are off-axis stiffness of a particular lamina (layer). There can be several laminae within a particular laminate. The fiber orientation of each lamina may or may not be along the coordinate axes x and y .

For each lamina, the on-axis stress–strain relations (along the fiber direction 1 and transverse to fiber direction 2) can be expressed as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 - \alpha_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T \\ \frac{1}{2} \gamma_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2} \gamma_{12} \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} \begin{bmatrix} -\alpha_1 \Delta T \\ -\alpha_2 \Delta T \\ 0 \end{bmatrix} \quad (1.16)$$

where Q_{ij} are the on-axis stiffness coefficients of a lamina, α_s are the coefficients of thermal expansion, and ΔT is the temperature change. These can be expressed in terms of the elastic constants of the lamina as follows:

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\
Q_{66} &= G_{12}
\end{aligned} \tag{1.17}$$

where E_1 and E_2 are the elastic moduli along the fiber direction and transverse to the fiber direction, respectively; G_{12} is the in-plane shear modulus; and ν_{12} is the major Poisson's ratio of the lamina.

There is also a reciprocity relation:

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \tag{1.18}$$

The elastic constants can be obtained by testing laminates made of unidirectional layers. One example of a set of elastic constants for carbon/epoxy is given in Table 1.1.

Table 1.1: Properties of a carbon/epoxy lamina [4].

E_1 (GPa)	155.0
E_2 (GPa)	12.10
G_{12} (GPa)	4.40
ν_{12}	0.248
α_1 ($10^{-6}/^\circ\text{C}$)	-0.018
α_2 ($10^{-6}/^\circ\text{C}$)	24.3
β_1 (%)	0
β_2 (%)	0.066
Thickness t (mm)	0.125

Note that the coefficient of thermal expansion here is different from that in eq. (1.5) due to the use of a different type of resin system.

The $\bar{Q}_{ij}$ in eq. (1.15) can be obtained from the Q_{ij} in eq. (1.17) by using the transformation equations:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{1.19}$$

and

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix} \quad (1.20)$$

where $m = \cos \theta$ and $n = \sin \theta$, and θ is the fiber angle.

Substituting eqs. (1.19) and (1.20) into eq. (1.16) yields

$$[T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} \begin{bmatrix} -\alpha_1 \Delta T \\ -\alpha_2 \Delta T \\ 0 \end{bmatrix} \quad (1.21)$$

which can be written as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix} + [T]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & 2Q_{66} \end{bmatrix} \begin{bmatrix} -\alpha_1 \Delta T \\ -\alpha_2 \Delta T \\ 0 \end{bmatrix} \quad (1.22)$$

which again can be written as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o + z k_x^o \\ \varepsilon_y^o + z k_y^o \\ \gamma_{xy}^o + z k_{xy}^o \end{bmatrix} + \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{bmatrix} -\alpha_x \Delta T \\ -\alpha_y \Delta T \\ -\alpha_{xy} \Delta T \end{bmatrix} \quad (1.23)$$

where

$$\overline{Q_{11}} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{22} \sin^4 \theta \quad (1.24a)$$

$$\overline{Q_{12}} = (Q_{11} + Q_{22} - 4Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{12} (\cos^4 \theta + \sin^4 \theta) \quad (1.24b)$$

$$\overline{Q_{16}} = (Q_{11} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos \theta \sin^3 \theta \quad (1.24c)$$

$$\overline{Q_{22}} = Q_{22} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{11} \sin^4 \theta \quad (1.24d)$$

$$\overline{Q_{26}} = (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \quad (1.24e)$$

$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{66} (\cos^4 \theta + \sin^4 \theta) \quad (1.24f)$$

and

$$\alpha_x = \alpha_1 m^2 + \alpha_2 n^2 \quad \alpha_y = \alpha_1 n^2 + \alpha_2 m^2 \quad \alpha_{xy} = 2(\alpha_1 - \alpha_2)mn \quad (1.25)$$

or

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \frac{1}{2}\alpha_{xy} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} = [T] \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix}$$

where α_1 and α_2 are on-axis coefficients of thermal expansion of a particular layer, as given in Table 1.1.

Laminate theory smears the individual layers and considers the laminate as a whole. As such, the lamina quantities in eq. (1.23) are integrated over the thickness to obtain a relation between the laminate quantities.

The stress quantities become the stress resultants and moment resultants as follows:

$$N_x = \int \sigma_x dz \quad N_y = \int \sigma_y dz \quad N_{xy} = \int \tau_{xy} dz \quad (1.26)$$

$$M_x = \int \sigma_x z dz \quad M_y = \int \sigma_y z dz \quad M_{xy} = \int \tau_{xy} z dz \quad (1.27)$$

The stiffness coefficients for the lamina become the stiffness coefficients for the laminate:

$$A_{ij} = \int \overline{Q}_{ij} dz \quad B_{ij} = \int \overline{Q}_{ij} z dz \quad D_{ij} = \int \overline{Q}_{ij} z^2 dz \quad (1.28)$$

The thermal stress resultants and thermal moment resultants are given as follows:

$$N_x^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{11}\alpha_x^T + \overline{Q}_{12}\alpha_y^T + \overline{Q}_{16}\alpha_{xy}^T \right) \Delta T dz \quad (1.29a)$$

$$N_y^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{12}\alpha_x^T + \overline{Q}_{22}\alpha_y^T + \overline{Q}_{26}\alpha_{xy}^T \right) \Delta T dz \quad (1.29b)$$

$$N_{xy}^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{16}\alpha_x^T + \overline{Q}_{26}\alpha_y^T + \overline{Q}_{66}\alpha_{xy}^T \right) \Delta T dz \quad (1.29c)$$

$$M_x^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{11}\alpha_x^T + \overline{Q}_{12}\alpha_y^T + \overline{Q}_{16}\alpha_{xy}^T \right) \Delta T z dz \quad (1.29d)$$

$$M_y^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{12} \alpha_x^T + \overline{Q}_{22} \alpha_y^T + \overline{Q}_{26} \alpha_{xy}^T \right) \Delta T z dz \quad (1.29e)$$

$$M_{xy}^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{16} \alpha_x^T + \overline{Q}_{26} \alpha_y^T + \overline{Q}_{66} \alpha_{xy}^T \right) \Delta T z dz \quad (1.29f)$$

With these definitions for the laminate quantities, and for the case where the only loading is due to thermal loads, the laminate equation can be written using the lamina equation (1.23) as follows:

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (1.30)$$

where the superscript T refers to temperature loading.

The inverse of eq. (1.30) can be written for the determination of the strains:

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \\ a_{12} & a_{22} & a_{26} & b_{21} & b_{22} & b_{26} \\ a_{16} & a_{26} & a_{66} & b_{61} & b_{62} & b_{66} \\ b_{11} & b_{21} & b_{61} & d_{11} & d_{12} & d_{16} \\ b_{12} & b_{22} & b_{62} & d_{12} & d_{22} & d_{26} \\ b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} \quad (1.31)$$

where the column on the left-hand side represents the in-plane strains and curvatures at the mid-plane of the laminate, the square matrix represents components of the compliance and is the inverse of the stiffness matrix, and the column on the right-hand side represents the thermal stress resultants (or shrinkage stress resultants) and thermal moment resultants (or shrinkage moment resultants), respectively.

1.5.2 Deformed shape

By obtaining the strains and curvatures as shown in eqs. (1.31), the deformation of the laminate due to cooling down from the curing temperature (or processing temperature) can be obtained as follows:

In eqs. (1.12)–(1.14), the curvatures are assumed to be constant (not functions of x and y). Integrating the first two equations results in two different expressions for $w^o(x, y)$.

$$w^o(x, y) = -\frac{1}{2}k_x^o x^2 + q(y)x + r(y) \quad (1.32a)$$

$$w^o(x, y) = -\frac{1}{2}k_y^o y^2 + s(x)y + t(x) \quad (1.32b)$$

where $q(y)$, $r(y)$, $s(x)$, and $t(x)$ are arbitrary functions of integration.

The twisting curvature is given by two expressions:

$$\frac{\partial^2 w^o(x, y)}{\partial x \partial y} = \frac{dq(y)}{dy} = -\frac{1}{2}k_{xy}^o \quad (1.33a)$$

$$\frac{\partial^2 w^o(x, y)}{\partial x \partial y} = \frac{ds(x)}{dx} = -\frac{1}{2}k_{xy}^o \quad (1.33b)$$

Each of these equations can be integrated to give

$$q(y) = -\frac{1}{2}k_{xy}^o y + K_1 \quad (1.34a)$$

$$s(x) = -\frac{1}{2}k_{xy}^o x + K_2 \quad (1.34b)$$

Using these, expressions (1.32a) and (1.32b) can be written as follows:

$$w^o(x, y) = -\frac{1}{2}k_x^o x^2 - \frac{1}{2}k_{xy}^o xy + K_1 x + r(y) \quad (1.35a)$$

$$w^o(x, y) = -\frac{1}{2}k_y^o y^2 - \frac{1}{2}k_{xy}^o xy + K_2 y + t(x) \quad (1.35b)$$

This leads to the conclusion that

$$r(y) = -\frac{1}{2}k_y^o y^2 + K_2 y + K_3 \quad (1.36a)$$

$$t(x) = -\frac{1}{2}k_x^o x^2 + K_1 x + K_3 \quad (1.36b)$$

and

$$w^o(x, y) = -\frac{1}{2} \left(k_x^o x^2 + k_y^o y^2 + k_{xy}^o xy \right) + K_1 x + K_2 y + K_3 \quad (1.37)$$

where the constants of integration associated with rigid body motions can be arbitrarily set to zero, resulting in

$$w^o(x, y) = -\frac{1}{2} \left(k_x^o x^2 + k_y^o y^2 + k_{xy}^o xy \right) \quad (1.38)$$

For the case of laminates made of layers at 0° and 90° only, that is, $[0/90]$ lay-up sequence, $k_{xy} = 0$, giving

$$w^o(x, y) = -\frac{1}{2} \left(k_x^o x^2 + k_y^o y^2 \right) \quad (1.39)$$

Equation (1.39) shows that the laminate has two curvatures, indicating a saddle shape. However, experimental results of many laminates show that only one curvature (either K_x or K_y) exists at room temperature after the laminate is cured and cooled to room temperature. Figure 1.8 shows an example sample. Hyer [7] also observed this. He used extended laminate theory to explain this behavior.

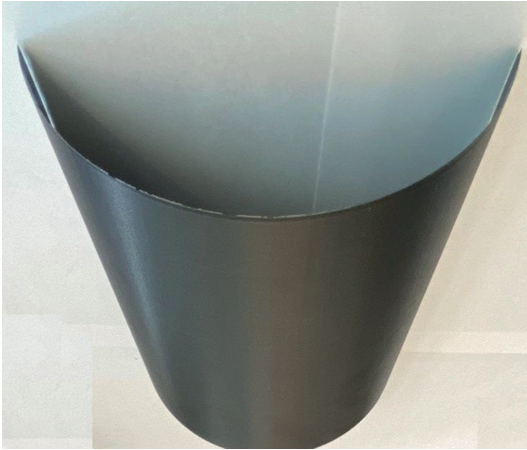


Figure 1.8: Shape of a $[0/90]$ laminate at room temperature.

1.6 Extended laminate theory by Hyer

Equation (1.39) shows that for the case of a laminate made with a lay-up sequence $[0/90]$, according to the classical laminate theory, there are two curvatures, K_x and K_y , with equal magnitude but opposite in sign. This means that the shape of the laminate is a saddle shape, with two equal and opposite curvatures. However, when such a laminate is made, after curing and cooling to room temperature, only one curvature is observed for square laminates, as shown in Figure 1.8. This is not a saddle shape, but a straight cylindrical shape. When a slight load is applied normal to the surface of the laminate, the laminate pops up and changes its curvature from one direction to the other (e.g., from K_x to K_y).

Hyer [7] also observed this behavior and attributed it to the nonlinear effect, which is not taken into account in the classical laminate theory. He developed an extended laminate theory to explain the phenomenon. The existence of the two room temperature shapes (i.e., the two cylinders) essentially ruled out a linear extension to the theory since a linear extension would lead to the prediction of a unique shape. Furthermore, since the out-of-plane deflections of the unsymmetric laminates are on the order of several laminate thicknesses, geometric nonlinearities were felt to have an important effect. Thus, in an effort to explain the behavior of unsymmetric laminates, classical laminate theory was extended to include geometric nonlinearities in the strain–displacement relationships. For laminates made of cross-ply layers, the linear strain–displacement relations in eqs. (1.9)–(1.11) are replaced with the following nonlinear relations:

$$\varepsilon_x^o = \frac{\delta u^o}{\delta x} + \frac{1}{2} \left(\frac{\delta w^o}{\delta x} \right)^2 \quad (1.40)$$

$$\varepsilon_y^o = \frac{\delta v^o}{\delta y} + \frac{1}{2} \left(\frac{\delta w^o}{\delta y} \right)^2 \quad (1.41)$$

$$\gamma_{xy}^o = \frac{\delta u^o}{\delta y} + \frac{\delta v^o}{\delta x} + \frac{\delta w^o}{\delta x} \frac{\delta w^o}{\delta y} \quad (1.42)$$

Equations (1.40)–(1.42) represent the main departure from classical laminate theory and include the usual approximations associated with thin plate theory when employing the nonlinear geometric effects in the strain–displacement relations. These approximations assume the elongation and shearing strains, and the squares of the rotations are of the same order of magnitude, and this order is small compared to unity.

Since it is assumed that external tractions are not considered during the cooling process, the total potential energy, including the effect of thermal expansion (or contraction), is given by

$$U = \int_{\text{Vol}} W dV \quad (1.43)$$

where, for the case of cross-ply laminates (with lay-up sequences $[0_m/90_n]$), W can be written in terms of the lamina stiffness coefficients as follows:

$$\begin{aligned} W = & \frac{1}{2} \overline{Q}_{11} \varepsilon_x^2 + \overline{Q}_{12} \varepsilon_x \varepsilon_y + \frac{1}{2} \overline{Q}_{66} \varepsilon_{xy}^2 + \frac{1}{2} \overline{Q}_{22} \varepsilon_y^2 - (\overline{Q}_{11} \alpha_x + \overline{Q}_{12} \alpha_y) \varepsilon_x \Delta T \\ & - (\overline{Q}_{12} \alpha_x + \overline{Q}_{22} \alpha_y) \varepsilon_y \Delta T \end{aligned} \quad (1.44)$$

The problem is reduced to finding the solutions for u° , v° , and w° as functions of x and y , which, through eqs. (1.6)–(1.8) and (1.40)–(1.42), minimize the energy given in eq. (1.43). Approximate solutions for u° , v° , and w° were sought. In seeking realistic approximate solutions for u° , v° , and w° , two assumptions are made:

- Even in attaining the cylindrical shape, the mid-plane elongation strains, ε_x° , and ε_y° , do not vary much from the linear prediction, i.e., ε_x° and ε_y° are independent of x and y .
- To the order of the nonlinearity considered here, the mid-plane shear strains ε_{xy}° are negligible.

With these assumptions, Hyer assumed functions for the in-plane displacements, then obtained the strains from these functions, and minimized the strain energy. He obtained solutions for the curvatures of square laminates ($L_x = L_y = L$) with lengths ranging from 0 to 150 mm per side. Two thicknesses were considered, $[0_2/90_2]_T$ and $[0_4/90_4]_T$. Experimental data were available for checking the results for these cases.

Figure 1.9 shows the result of the thinner laminate, and Figure 1.10 shows the result for the thicker plate. A few observations can be made from these figures:

1. The upper part of each figure shows the curvature situation along the x -direction, while the lower part of each figure shows the curvature situation along the y -direction.
2. Point A represents the curvature for the case of small length (almost 0). This represents the result from the classical laminate theory. The curvatures are of equal and opposite signs, showing a saddle shape configuration.
3. As the length increases, the curvature decreases in magnitude and follows the path AB . The saddle shape configuration still remains.
4. As the length increases from that corresponding to point B, the laminate curvature has multiple values. The path can follow either BC , BD , or BE . It was indicated by Hyer that path BD (representing the saddle situation) is unstable. As such, either path BC or BE is possible. It is shown in the two figures that if path BC is followed, then curvature along the x direction is dominant, while there is a little curvature along the y direction. On the other hand, if path BE is followed, then curvature along the y direction is dominant, while there is little curvature along the x direction. Whether path BC or BE is followed depends on the relative length along the two directions and the existence of defects, such as random distribution of fibers and variation of fiber orientations, that are inherent within the composite material.
5. Beyond a certain length (in the case of square laminates), either path BC or BE shows that the value of the curvature is approximately the same as the curvature at point A, which is the curvature calculated by laminate theory. This is an important point because for relatively large laminates (with high length/thickness ratios), the curvature of the laminate at room temperature can be obtained by just performing the classical laminate theory calculations. This simplifies the analysis.

6. The value of the curvature either at point C or E is about $6/m$ for thin laminates $[0_2/90_2]$, whereas that for the thick laminate $[0_4/90_4]$ is about $3/m$. As such, the curvature for thin laminates is larger than that for thick laminates. The ratio between the curvatures is the same as the ratio between the thicknesses (for laminates of the same lateral dimensions and made of the same materials). This point will be shown mathematically later.
7. The understanding of the change in the shape of the laminate from one configuration to another for laminates of different lateral dimensions is important for the determination of the lay-up sequences to fit a certain desired shape, which is essential for the method of 4DPCs.

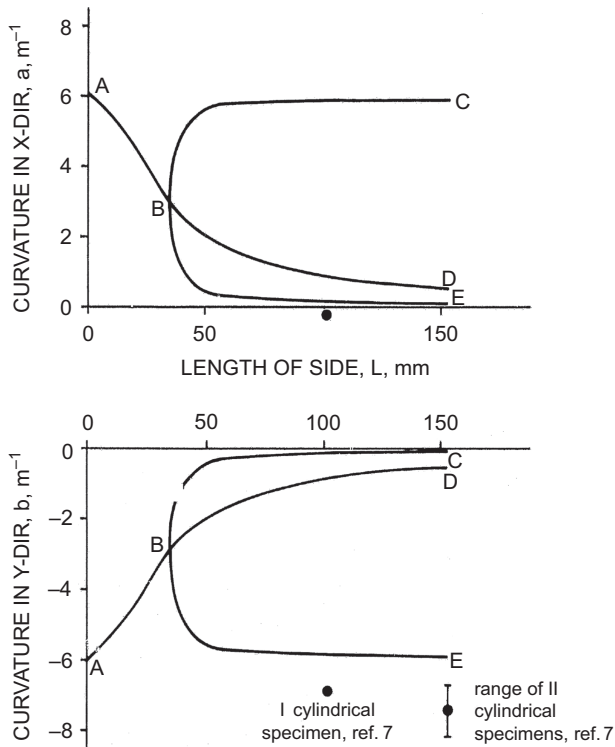


Figure 1.9: Distribution of curvatures for a square plate $[0_2/90_2]$ made of carbon/epoxy T300/5208. Reference [7] in the figure is for the experimental result (Hyer M.W., "Some observations on the cured shape of thin unsymmetric laminates", J. Composite Materials, Vol. 15, March 1981, p. 175).

Observation no. 6 above mentioned that the radius of curvature of laminates with a sufficiently low t/a ratio (where a is the side length and t is the thickness of the laminate) would have a radius of curvature with value to be about the same as that calculated using the laminate theory. As such, equations for the radius of curvature of a few simple

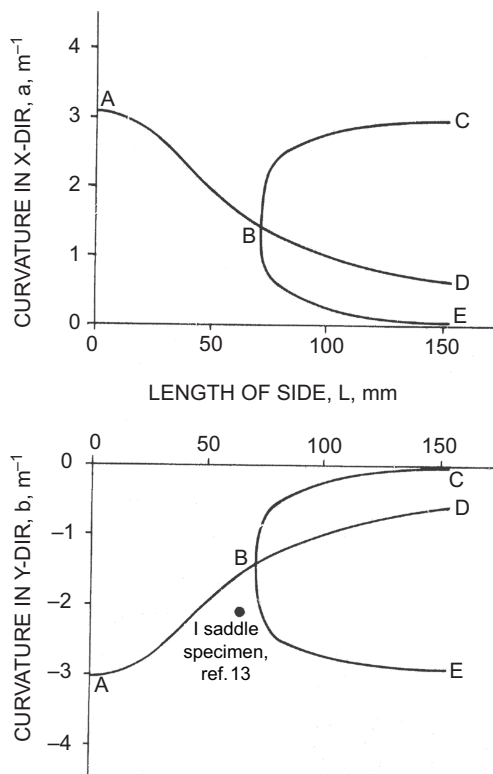


Figure 1.10: Distribution of curvatures for a square plate $[0_4/90_4]$ made of carbon/epoxy T300/5208. Reference [13] in the figure is for the experimental result (Pagano, N.J., and Hahn, H.T., “Evaluation of composite curing stresses”, *Composite Materials, Testing and Design*, 4th conference, ASTM STP 617, 1977, pp. 317–329).

laminates (those made of 0° and 90° layers) will be derived in Section 1.8. These equations will help to understand the effect of different parameters on the radius of curvature.

1.7 Physical explanation for the occurrence of different shapes

As shown in the above section, Hyer [7] used the extended laminate theory to show that as the length of the laminate increases, there is a certain transition point called the bifurcation point where the saddle shape changes to a cylindrical bending shape either along the x direction or along the y direction. This has shed light on the possibility of the different shapes, but it did not explain why there is a change. The reason is given below.

For a square laminate with a lay-up sequence $[0/90]$, as shown in Figure 1.11a, the bending possibility due to asymmetry along either the x direction or the y direction is equal to each other. Laminate theory assumes this equal competition. As such, laminate theory predicts a saddle shape, as shown in Figure 1.11b, regardless of the dimensions of

the laminate. However, in reality, there are differences along the x direction compared to the y direction. This difference arises from the variability in the microstructure of the materials. Continuous fiber composites are made of millions of fibers with tiny diameters. These are bonded by thin layers of the matrix material. There are variations in the fiber distribution, fiber volume fraction, and the degree of cure of the resin from point to point. These variations create differences in material properties along the x and y directions. The amount of bending moment due to thermal load increases with the lateral dimension of the laminate. For small-sized laminates, the thermal bending moment is not large enough to trigger the effect of the difference. However, when the size of the laminate is large enough that the thermal bending is sufficient to trigger the effect, one direction becomes dominant over the other. When that happens, bending along one direction is favored over the other direction. As the size increases, the dominance grows, and the cylindrical bending shape becomes the one observed. When that happens, the shape will be that of Figure 1.11c or d, depending on which side is more dominant. The point at which the transition occurs is called the bifurcation point.

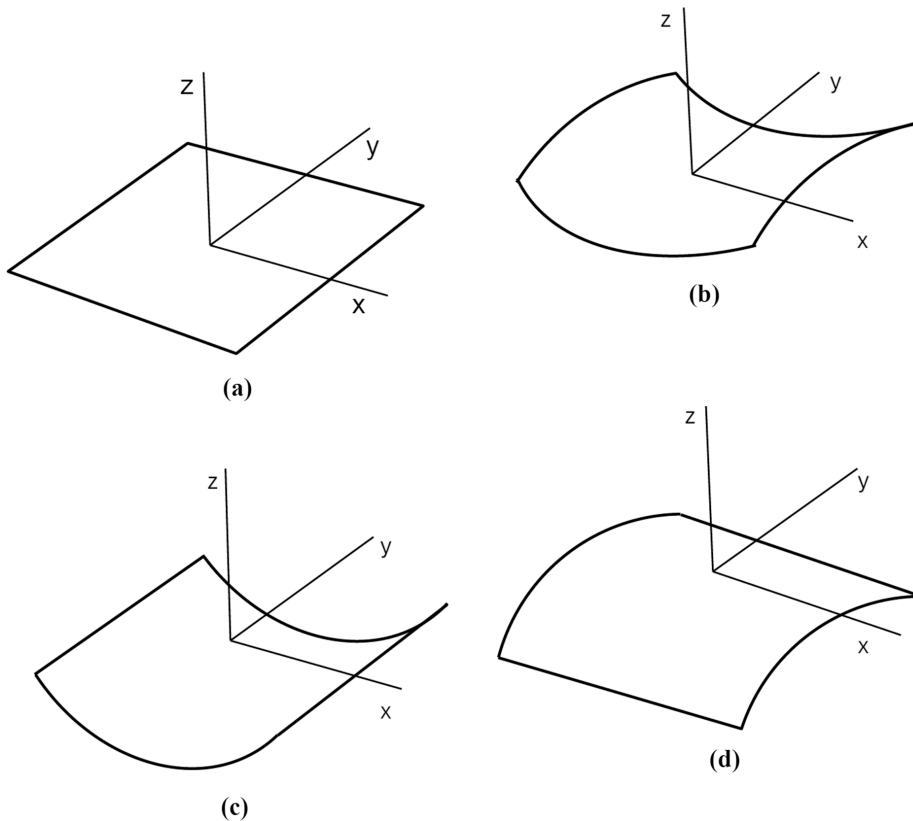


Figure 1.11: Possibilities of different shapes.

1.8 Laminates made of lay-up sequences $[0_{2m}/90_{2n}]$

For the case of laminates made of the lay-up sequences $[0_{2m}/90_{2n}]$, the fiber angles θ are either 0° or 90° , and the above equations can be simplified. The simplified versions of these equations can be used to extract useful properties for the unsymmetric laminates.

1.8.1 For 0° layers

Equation (1.24) becomes

$$\begin{aligned}\overline{Q_{11}} &= Q_{11} \\ \overline{Q_{12}} &= Q_{12} \\ \overline{Q_{16}} &= 0 \\ \overline{Q_{22}} &= Q_{22} \\ \overline{Q_{26}} &= 0 \\ \overline{Q_{66}} &= Q_{66}\end{aligned}\tag{1.45}$$

Equation (1.25) becomes

$$\alpha_x = \alpha_1 \quad \alpha_y = \alpha_2 \quad \alpha_{xy} = 0\tag{1.46}$$

1.8.2 For 90° layers

Equation (1.24) becomes

$$\begin{aligned}\overline{Q_{11}} &= Q_{22} \\ \overline{Q_{12}} &= Q_{12} \\ \overline{Q_{16}} &= 0 \\ \overline{Q_{22}} &= Q_{11} \\ \overline{Q_{26}} &= 0 \\ \overline{Q_{66}} &= Q_{66}\end{aligned}\tag{1.47}$$

Equation (1.25) becomes

$$\alpha_x = \alpha_2 \quad \alpha_y = \alpha_1 \quad \alpha_{xy} = 0\tag{1.48}$$

The coefficients of A_{ij} , B_{ij} , and D_{ij} matrices can be written as follows (h is the thickness of a layer and H is the thickness of the laminate):

$$[A] = \begin{bmatrix} 2h(Q_{11}m + Q_{22}n) & 2hQ_{12}(m + n) & 0 \\ 2hQ_{12}(m + n) & 2h(Q_{11}n + Q_{22}m) & 0 \\ 0 & 0 & 2hQ_{66}(m + n) \end{bmatrix} \quad (1.49)$$

$$[B] = \begin{bmatrix} 2mnh^2(Q_{22} - Q_{11}) & 0 & 0 \\ 0 & 2mnh^2(Q_{11} - Q_{22}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.50)$$

$$[D] = \begin{bmatrix} \frac{h^3}{3} [Q_{11}(2m^3 + 6mn^2) + Q_{22}(2n^3 + 6m^2n)] & & \\ \frac{2h^3}{3} Q_{12}(m + n)^3 & & \\ 0 & & \\ \frac{2h^3}{3} Q_{12}(m + n)^3 & 0 & \\ \frac{h^3}{3} [Q_{11}(2n^3 + 6m^2n) + Q_{22}(2m^3 + 6mn^2)] & 0 & \\ 0 & \frac{2h^3}{3} Q_{66}(m + n)^3 & \end{bmatrix} \quad (1.51)$$

From eq. (1.29), we have

$$N_x^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q_{11}}\alpha_x^T + \overline{Q_{12}}\alpha_y^T + \overline{Q_{16}}\alpha_{xy}^T \right) \Delta T dz = 2\Delta Th [Q_{11}\alpha_1 + Q_{12}\alpha_2]m + (Q_{22}\alpha_2 + Q_{12}\alpha_1)n \quad (1.52)$$

$$N_y^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q_{12}}\alpha_x^T + \overline{Q_{22}}\alpha_y^T + \overline{Q_{26}}\alpha_{xy}^T \right) \Delta T dz = 2\Delta Th [Q_{12}\alpha_1 + Q_{22}\alpha_2]m + (Q_{12}\alpha_2 + Q_{11}\alpha_1)n \quad (1.53)$$

$$N_{xy}^T = 0 \quad (1.54)$$

$$M_x^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q_{11}}\alpha_x^T + \overline{Q_{12}}\alpha_y^T + \overline{Q_{16}}\alpha_{xy}^T \right) \Delta T z dz = 2mnh^2\Delta T [-(Q_{11}\alpha_1 + Q_{12}\alpha_2) + (Q_{22}\alpha_2 + Q_{12}\alpha_1)] \quad (1.55)$$

$$M_y^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q_{12}}\alpha_x^T + \overline{Q_{22}}\alpha_y^T + \overline{Q_{26}}\alpha_{xy}^T \right) \Delta T z dz = 2mnh^2\Delta T [-(Q_{12}\alpha_1 + Q_{22}\alpha_2) + (Q_{12}\alpha_2 + Q_{11}\alpha_1)] \quad (1.56)$$

$$M_{xy}^T = 0 \quad (1.57)$$

1.8.3 For the special case where $m = n = p$

$$[A] = \begin{bmatrix} 2hp(Q_{11} + Q_{22}) & 4hpQ_{12} & 0 \\ 4hpQ_{12} & 2hp(Q_{11} + Q_{22}) & 0 \\ 0 & 0 & 4hQ_{66}p \end{bmatrix} \quad (1.58)$$

$$[B] = \begin{bmatrix} 2h^2p^2(Q_{22} - Q_{11}) & 0 & 0 \\ 0 & 2h^2p^2(Q_{11} - Q_{22}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.59)$$

$$[D] = \begin{bmatrix} \frac{8h^3}{3}p^3(Q_{11} + Q_{22}) & \frac{16h^3p^3}{3}Q_{12} & 0 \\ \frac{16h^3p^3}{3}Q_{12} & \frac{8h^3}{3}p^3(Q_{11} + Q_{22}) & 0 \\ 0 & 0 & \frac{16h^3p^3}{3}Q_{66} \end{bmatrix} \quad (1.60)$$

$$N_x^T = 2\Delta Thp[Q_{11}\alpha_1 + Q_{12}\alpha_2 + Q_{22}\alpha_2 + Q_{12}\alpha_1] \quad (1.61)$$

$$N_y^T = 2\Delta Thp[Q_{12}\alpha_1 + Q_{22}\alpha_2 + Q_{12}\alpha_2 + Q_{11}\alpha_1] \quad (1.62)$$

$$N_{xy}^T = 0 \quad (1.63)$$

$$M_x^T = 2p^2h^2\Delta T[-(Q_{11}\alpha_1 + Q_{12}\alpha_2) + (Q_{22}\alpha_2 + Q_{12}\alpha_1)] \quad (1.64)$$

$$M_y^T = 2p^2h^2\Delta T[-(Q_{12}\alpha_1 + Q_{22}\alpha_2) + (Q_{12}\alpha_2 + Q_{11}\alpha_1)] \quad (1.65)$$

$$M_{xy}^T = 0 \quad (1.66)$$

1.8.4 In eqs. (1.58)–(1.65), if $p = 0.5$ (the case of the [0/90] laminate)

$$[A] = \begin{bmatrix} h(Q_{11} + Q_{22}) & 2hQ_{12} & 0 \\ 2hQ_{12} & h(Q_{11} + Q_{22}) & 0 \\ 0 & 0 & 2hQ_{66} \end{bmatrix} \quad (1.67)$$

$$[B] = \begin{bmatrix} \frac{h^2}{2}(Q_{22} - Q_{11}) & 0 & 0 \\ 0 & \frac{h^2}{2}(Q_{11} - Q_{22}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.68)$$

$$[D] = \begin{bmatrix} \frac{h^3}{3}(Q_{11} + Q_{22}) & \frac{2h^3}{3}Q_{12} & 0 \\ \frac{2h^3}{3}Q_{12} & \frac{h^3}{3}(Q_{11} + Q_{22}) & 0 \\ 0 & 0 & \frac{2h^3}{3}Q_{66} \end{bmatrix} \quad (1.69)$$

The thermal stress and moment resultants can be written as follows:

$$N_x^T = N_y^T = \Delta T h [Q_{11}\alpha_1 + Q_{12}\alpha_2 + Q_{22}\alpha_2 + Q_{12}\alpha_1] = \Delta T h [(Q_{11} + Q_{12})\alpha_1 + (Q_{22} + Q_{12})\alpha_2] \quad (1.70)$$

$$N_{xy}^T = 0 \quad (1.71)$$

$$M_x^T = -M_y^T = \frac{1}{2}h^2\Delta T [(Q_{12}\alpha_1 + Q_{22}\alpha_2) - (Q_{12}\alpha_2 + Q_{11}\alpha_1)] = \frac{1}{2}h^2\Delta T [(Q_{12} - Q_{11})\alpha_1 + (Q_{22} - Q_{12})\alpha_2] \quad (1.72)$$

$$M_{xy}^T = 0 \quad (1.73)$$

By substituting eqs. (1.49)–(1.57) into eqs. (1.30), the strains and curvature resulting from the temperature difference of $\Delta T = -157^\circ\text{C}$ (and neglecting the effect of cure shrinkage) for the case of a $[0/90]$ laminate can be obtained.

Equation (1.30) can be simplified to

$$\begin{bmatrix} N_x^T \\ N_y^T \\ M_x^T \\ M_y^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} \\ A_{12} & A_{22} & B_{12} & B_{22} \\ B_{11} & B_{12} & D_{11} & D_{12} \\ B_{12} & B_{22} & D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ k_x^o \\ k_y^o \end{bmatrix} \quad (1.74)$$

For the case of the $[0/90]$ laminate, the following relations can be obtained:

$$N_x^T = N_y^T \quad M_x^T = -M_y^T \quad (1.75)$$

$$A_{11} = A_{22} \quad B_{11} = -B_{22} \quad B_{12} = 0 \quad D_{11} = D_{22} \quad (1.76)$$

The four equations in (1.74) can be written separately as follows:

The first and second equations are

$$N_x^T = A_{11}\varepsilon_x^o + A_{12}\varepsilon_y^o + B_{11}k_x^o \quad (1.77)$$

$$N_y^T = A_{12}\varepsilon_x^o + A_{22}\varepsilon_y^o + B_{22}k_y^o \quad (1.78)$$

Subtracting (1.78) from (1.77) and rearranging give

$$\varepsilon_x^o - \varepsilon_y^o = -\frac{B_{11}(k_x^o + k_y^o)}{A_{11} - A_{12}} \quad (1.79)$$

The third equation of (1.69) is

$$M_x^T = B_{11}\varepsilon_x^o + D_{11}k_x^o + D_{12}k_y^o \quad (1.80)$$

which can be written as

$$\varepsilon_x^o = \frac{M_x^T - D_{11}k_x^o - D_{12}k_y^o}{B_{11}} \quad (1.81)$$

Similarly, from the fourth equation of (1.74)

$$\varepsilon_y^o = \frac{M_y^T - D_{12}k_x^o - D_{22}k_y^o}{B_{22}} \quad (1.82)$$

which can be written as

$$\varepsilon_y^o = \frac{M_x^T + D_{12}k_x^o + D_{22}k_y^o}{B_{11}} \quad (1.83)$$

Using (1.81) and (1.83) in (1.79) and rearranging terms give

$$k_x^o = -k_y^o \quad (1.84)$$

Using this in eq. (1.79) again gives

$$\varepsilon_x^o = \varepsilon_y^o \quad (1.85)$$

Using (1.84) and (1.85) in (1.77) and (1.80) gives

$$N_x^T = (A_{11} + A_{12})\varepsilon_x^o + B_{11}k_x^o \quad (1.86)$$

$$M_x^T = B_{11}\varepsilon_x^o + (D_{11} - D_{12})k_x^o \quad (1.87)$$

Solving (1.81) and (1.82) gives

$$k_x^o = \frac{N_x^T B_{11} - M_x^T (A_{11} + A_{12})}{B_{11}^2 - (D_{11} - D_{12})(A_{11} + A_{12})} \quad (1.88)$$

and

$$\varepsilon_x^o = \frac{N_x^T (D_{11} - D_{12}) - B_{11} M_x^T}{(A_{11} + A_{12})(D_{11} - D_{12}) - B_{11}^2} \quad (1.89)$$

Substituting expressions (1.67)–(1.73) into eq. (1.88) gives

$$k_x^o = \frac{\text{Numerator}}{\text{Denominator}} \quad (1.90)$$

$$\begin{aligned} \text{Numerator} = & \Delta T h [Q_{11} \alpha_1 + Q_{12} \alpha_2 + Q_{22} \alpha_2 + Q_{12} \alpha_1] \frac{h^2}{2} (Q_{11} + Q_{22}) \\ & - \frac{1}{2} h^2 \Delta T [(Q_{12} \alpha_1 + Q_{22} \alpha_2) - (Q_{12} \alpha_2 + Q_{11} \alpha_1)] [h(Q_{11} + Q_{22}) + 2hQ_{12}] \end{aligned} \quad (1.91)$$

$$\text{Denominator} = \frac{h^4}{4} (Q_{22} - Q_{11})^2 - \left[\frac{h^3}{3} (Q_{11} + Q_{22}) - \frac{2h^3}{3} Q_{12} \right] [h(Q_{11} + Q_{22}) + 2hQ_{12}] \quad (1.92)$$

From (1.90)–(1.92), it can be seen that the curvature is inversely proportional to h . Larger h results in smaller curvature (or larger radius of curvature). For example, a laminate with a lay-up sequence $[0_2/90_2]$ would have an h value that is twice as large as a laminate with a lay-up sequence $[0/90]$.

In eq. (1.67), h means the thickness of a single layer. However, in eqs. (1.90)–(1.92), h can also have the meaning of a ply group. For example, 0_2 is a ply group, and 90_2 is another ply group in the $[0_2/90_2]$ laminate. The radius of curvature of laminate $[0_2/90_2]$ would be two times as large as the radius of curvature for the $[0/90]$ laminate. This means that thinner laminates tend to be more curvy than thicker laminates.

1.8.5 Experiments

Experiments were carried out to make laminates with a stacking sequence of $[0/90]$. Figure 1.12 shows the flat lay-up configuration. Figure 1.8 shows a photograph of the sample. It can be seen that the final curved shape has a cylindrical configuration.

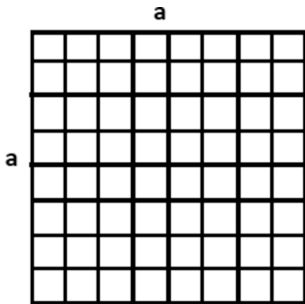


Figure 1.12: Configuration of square laminates with a $[0/90]$ stacking sequence.

Table 1.2 shows the strains and curvatures obtained for the $[0/90]$ laminate determined using the laminate equations above and the properties from Table 1.1. In this table, the radius of curvature is the inverse of the curvature (i.e., $R = 1/K$). It can be

seen that the laminate theory can be used to determine the curvatures of the $[0/90]$ laminates. The same statement can be extended to the case of $[0_{2m}/90_{2n}]$ laminates.

Table 1.2: Strains and curvatures and radii for laminates with stacking sequences $[0/90]$, using properties from Table 1.1.

ϵ_x	-0.0018
ϵ_y	-0.0018
ϵ_{xy}	0
K_x (1/m)	-22.76
K_y (1/m)	22.76
K_{xy} (1/m)	0
R_x (mm)	-43.9
R_y (mm)	43.9

1.9 Effect of geometrical dimensions on the bifurcation point

Figures 1.9 and 1.10 show that there exists a point B, which corresponds to the transition from a saddle shape to a cylindrical shape, depending on the geometry of the laminate. For laminates with the shape of a square in the flat configuration, if the length is less than a value corresponding to point B, the laminate will take the shape of a saddle after the laminate is cured and cooled to room temperature. At lengths above the value corresponding to point B, the laminate will take a cylindrical shape. Point B is called the bifurcation point. It is necessary to understand the effect of geometrical quantities on the bifurcation point in order to provide guidelines for the design of flat laminates to achieve a certain curved configuration. It should be noted that the thickness of the laminate stays the same as the length of the laminate varies in Figures 1.9 and 1.10. As such, for larger lengths, the thickness over side length ratio t/a would be less and vice versa. The effect of the thickness over side length ratio (t/a) for square laminates with the properties given in Table 1.1 is illustrated in Figure 1.13. The results were obtained using both extended laminate theory and finite element method. It can be seen that the bifurcation point varies nonlinearly with the ratio t/a . The bifurcation point is around room temperature at a t/a value of about 0.012. This means that for thick laminates, where the thickness is more than 1.2% of the side length, a saddle state can appear at room temperature. For a laminate with a thickness of 3 mm, a saddle configuration can occur for laminates with a side length of less than 250 mm (9.84 inches).

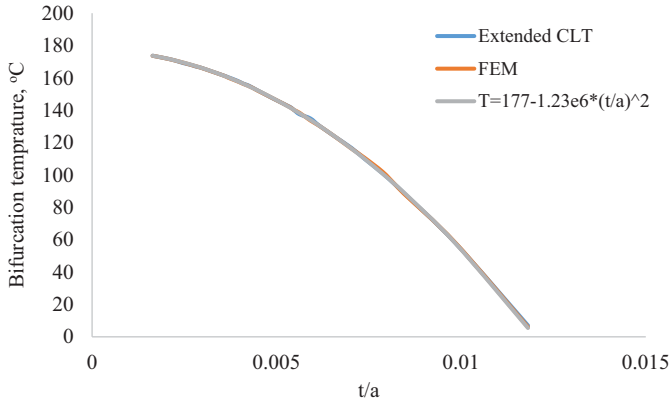


Figure 1.13: Effect of thickness-to-length ratio (t/a) for square laminates with properties listed in Table 1.1 [8].

1.9.1 Finite element simulation to illustrate the change in shape

Fakhimi and Hoa [8] used the extended laminate theory and the finite element method to determine the effect of geometrical parameters on the bifurcation point. It was shown that as a square cured laminate is cooled from cure temperature to room temperature, the flat configuration would undergo a change in shape from flat to saddle configuration, and then to cylindrical configuration, as shown in Figure 1.14. In the finite element analysis, the plate is modeled with a mesh of elements. For the case where there is perfect similarity between the x and y directions, the fixed point in the model should be at the center of the plate. However, in order to take into account the variability of the materials that gives rise to the dominance of one direction over the other, the fixed point is shifted toward one node along one side.

The initial condition of the plate (laminate) is when the stack of prepregs is placed in an autoclave (or oven), it is subjected to the cure cycle. After being cured at 177 °C, the laminate was flat. Nonlinear finite element analysis was used to determine the deformation of the laminate as it was being cooled. An incremental approach was used where the temperature was changed by -1 °C in each increment. The dimensions of the deformed structure at the end of one increment were used as input for the analysis of the next increment. Figure 1.14a shows the configuration of the laminate after one increment (i.e., at 176 °C). It can be seen that the shape of the laminate has a saddle shape, with equal deformation along the x and y directions. Figure 1.14b shows the configuration after two increments (i.e., at 175 °C). The saddle configuration still remains at this temperature. Figure 1.14c and d shows the beginning of the formation of the cylindrical shape, with some decreasing appearance of the saddle shape. Figure 1.14e shows the configuration after the 10th increment (i.e., at 167 °C). At this

temperature, the shape is completely cylindrical. This shows that the bifurcation point occurs at around this temperature. Figure 1.14f shows the configuration at room temperature, which of course has the cylindrical shape.

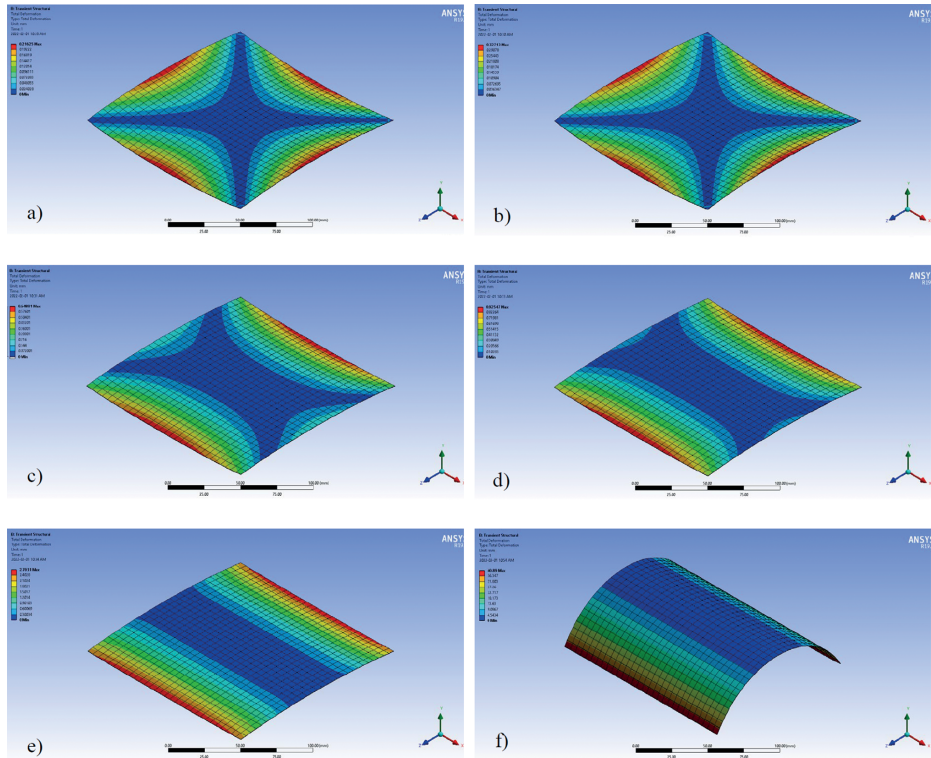


Figure 1.14: Change in shape of a flat laminate as it is being cooled from cure temperature to room temperature – 12 mm × 12 mm, [0/90] carbon/epoxy [8]: (a) 176 °C, (b) 175 °C, (c) 174 °C, (d) 173 °C, (e) 167 °C, and (f) 20 °C.

1.10 Conclusions

In this chapter, an introduction to composite materials and their use in engineering applications, along with the laminate theory, which can be used to determine shapes of simple laminates (such as [0/90] laminates), have been presented. The existence of the bifurcation point where the shape changes from saddle to cylindrical is also presented. A physical explanation for the change in shapes is also provided.

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Chapter 2

Differences between regular 4D printing (4DP) and 4D printing of composites (4DPC)

There exists a technique called 4D printing (abbreviated here as regular 4DP). There are many publications on this technology, for example [1–8]. This technique depends on the constituents of the materials to enable the change in shape of the structure from flat to curved. A few examples of these are shape-memory alloys or shape-memory polymers.

It is important to distinguish between regular 4DP and 4D printing of composites (4DPC). The similarity between 4DP and 4DPC lies in the concept. In both techniques, a flat structure changes its shape from flat to curved due to a stimulus from an external activation source such as exposure to heating (or cooling), light, electricity, magnetic field, or moisture. The materials used in 4DP are mostly special types of plastic. Sometimes a small amount of short fibers can be incorporated into the plastics to enhance their mechanical properties. Sometimes in the 4DP processes, the materials are also called composites, which can make things confusing. However, there are large differences between 4DP and 4DPC. These are outlined below.

2.1 Differences between short, discontinuous fibers and long, continuous fibers

The materials used in 4DP are mostly thermoplastic polymers, and sometimes these plastics may be reinforced with short, discontinuous fibers. The reason for this is that the method for material deposition in 4DP is 3D printing, particularly fused deposition modeling (FDM), where beads of the plastics (reinforced or not) are squeezed out of a nozzle and deposited onto the surface of a flat substrate. Figure 2.1 shows a schematic of this process. The materials used in 4DP are usually thermoplastic resins, such as polylactic acid (PLA) or nylon. The mechanical properties of these materials usually have small values. The modulus of the material is on the order of 20 GPa, and the strength is on the order of 50 MPa. During manufacturing, the soft plastic in the form of filament is fed into the deposition head. The deposition head heats and melts the plastic. The melted plastic is deposited on a mandrel (flat or curved). Different beads of the melted plastic are deposited side by side and on top of each other. Usually, there is no compression to compact the beads together. As such, voids usually exist. The lack of compaction does not provide good mechanical properties. As such, structures made using FDM usually have the shape but do not have high stiffness and strength.

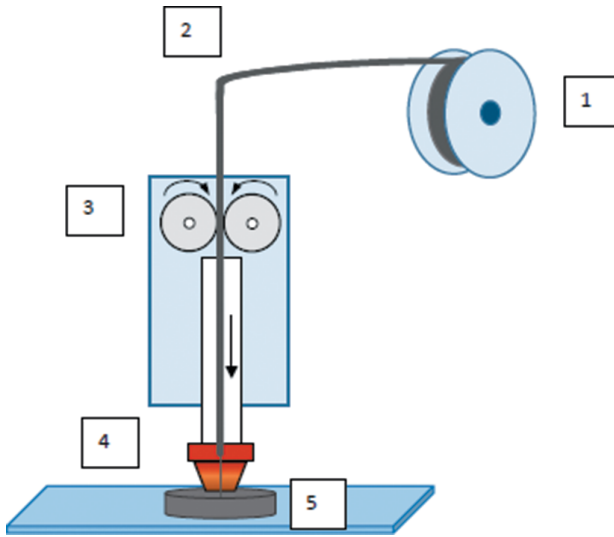


Figure 2.1: Schematic of fused deposition modeling in 3D printing: 1. material creel, 2. roller, 3. take-up rollers, 4. deposition head, and 5. deposited material.

The materials used in 4DPC are thermoset resin (e.g., epoxy) reinforced with long, continuous fibers. It is important to distinguish between thermoplastic resin and thermoset resin. Thermoplastic resins (used in 4DP) usually have long molecules. The softening and hardening of thermoplastic resin depend only on the temperature. At high temperatures, the molecules of the resin can move relative to each other easily. The viscosity of the resin is reduced, and it can flow. At low temperatures, it is difficult for the molecules to move relative to each other. Its viscosity is high, and it becomes solid. For thermoset resin (used in 4DPC), the molecules are usually short. As such, the viscosity of the resin is low at room temperature. The resin becomes solid due to the chemical reaction between the molecules of the resin and a secondary set of molecules called the linking molecules. Due to the basic differences in the molecular setup, the manufacturing principles of composites made of thermoplastic resin and thermoset resin are very different. Thermoplastic resin can be softened and hardened many times by heating and cooling. On the other hand, for thermoset resin, once it is hardened, it cannot be softened again by heating. While it is relatively easy to do 4DPC using thermoset resin, it is very difficult to do 4DPC with thermoplastic resin.

In 4DPC, the resin, at the start of the process, is partially cured such that the combined long fiber and resin form is called “prepregs.” Figure 2.2 shows a roll of the prepreg material of carbon/epoxy. The sheet of prepreg has a thickness similar to that of a sheet of paper (between 0.125 and 0.150 mm). The prepreg contains long continuous fibers embedded in uncured (or partially cured) resin. Due to the long continuous fibers, after the resin is cured, the composite material has very high mechanical prop-

erties. The modulus along the fiber direction is about 180 GPa, while the strength along the same direction is about 1,000 MPa. It can be seen that the material used in 4DPC has a modulus that is about 10 times as large as the material used in 4DP, and the strength is about 20 times as large. Due to this, structures manufactured using 4DPC can resist much larger loads compared to those made using 4DP.

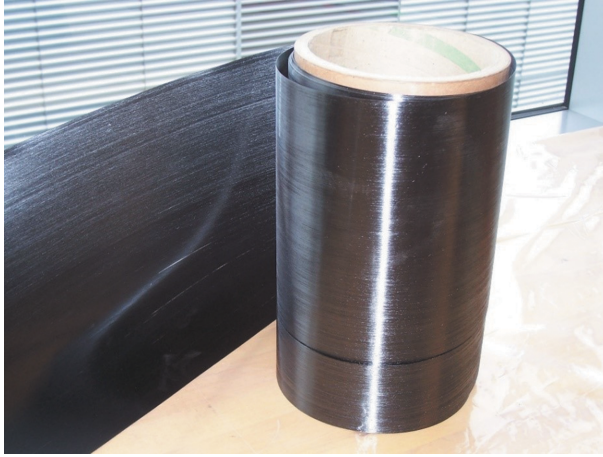


Figure 2.2: A roll of preregs made of carbon/epoxy.

2.2 Differences in the method of deposition of materials

In regular 4DP, the materials are deposited using 3D printers. The materials in the form of melts are squeezed through small orifices and deposited onto flat surfaces. The motion of the deposition head is controlled by a computer.

In 4DPCs, the materials can be deposited in one of two ways: hand lay-up (Figure 2.3) and by using an automated fiber placement (AFP) machine (Figure 2.4). For simple structures requiring only straight fibers, both methods can be used. For structures requiring curved fibers, only AFP can be used. In hand lay-up, sheets of preregs are deposited by hand over the surface of a flat tool. Each sheet forms a layer, and each layer has fibers oriented in a certain direction. The fiber orientation can vary from layer to layer. In AFP, strips of preregs are deposited one strip at a time on top of the flat (or curved) tool. The fiber orientation in one strip may differ from the fiber orientation in other strips. The ensemble of the strips on a certain elevation forms a layer.

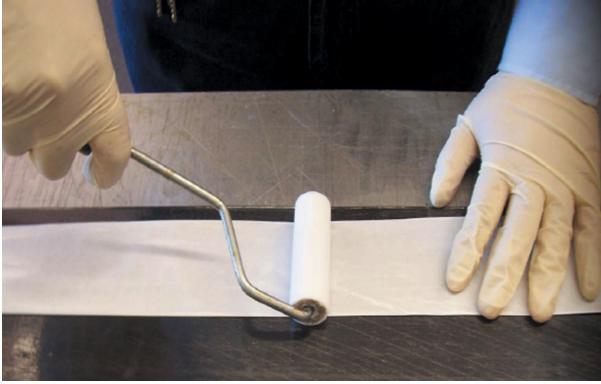


Figure 2.3: Hand lay-up method.

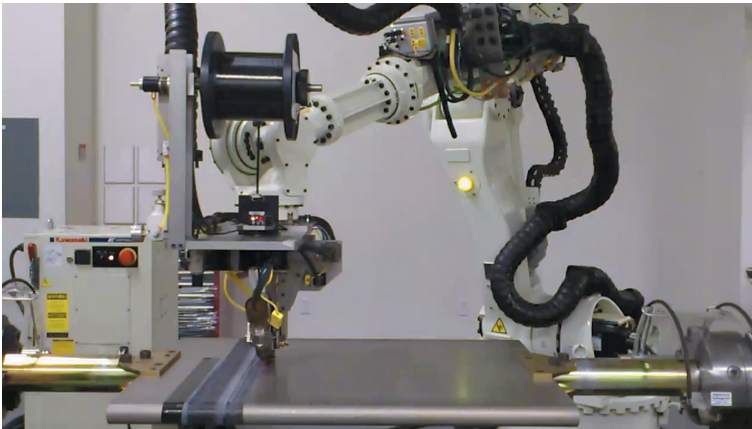


Figure 2.4: An automated fiber placement (AFP) machine.

2.3 Secondary processes

In 4DP, the material used is usually thermoplastic. Usually, there is no secondary process. The product as produced by the 3D printer is final. In 4DPC, the material is a thermoset (e.g., epoxy). As such, the stack of deposited prepreps needs to be cured using either an autoclave or an oven. The curing temperature depends on the type of resin and can be room temperature or elevated temperature. Most epoxy resin used for aircraft structures is cured at about 177 °C. The secondary process is an additional process, but it provides an opportunity to enhance the quality of the laminates.

2.4 The mechanism for the change of shape

2.4.1 For 4DP

In 4DP, the mechanism for the change of shape from flat to curved depends on the special properties of the material. The flat structure can either be made of different materials, or the material itself has special characteristics that allow the change in shape when an external stimulus is applied.

2.4.1.1 Deposition of different materials at different positions

Figure 2.5 shows the spatial arrangement of the different materials used to make a curved structure. Figure 2.5a illustrates the procedure for material deposition. There are three different types of materials in the structure. Material 1 (e.g., portion *AB*) is a regular plastic material that constitutes the main part of the structure. Materials 2 and 3 (e.g., portion *BC*) form the links between segments of material 1. When subjected to an external stimulus, such as water absorption or heat, material 2 will expand more than material 3. The combination of materials 2 and 3 in the link will cause the link to curl up upon exposure to the external stimulus. Figure 2.5b shows the final configuration of the structure upon exposure to the external stimulus.

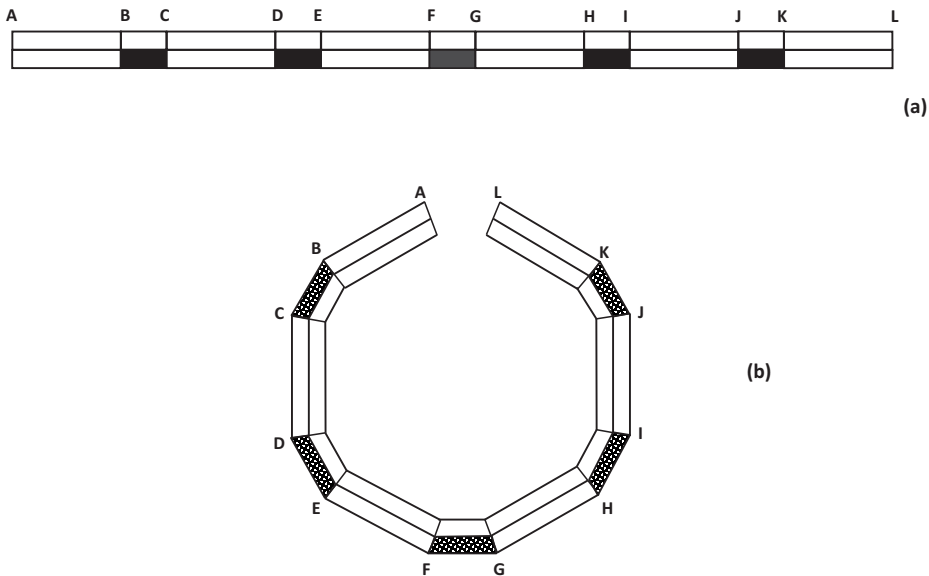


Figure 2.5: Different materials are used at different positions within the structure: (a) deposited material in the flat configuration and (b) curled configuration when subjected to an external stimulus.

2.4.1.2 Stimuli response polymers

Some methods in 4DP utilize materials that respond to external stimuli. The following are examples of materials that possess special properties [2]:

- Materials that change shape due to heating (or cooling): Shape-memory polymers (polyurethane), PLA, polycaprolactone, gelatin, and agarose.
- Material that changes shape due to enzymes: Silk fibroin
- Materials that change shape due to a change in pH value: Chitosan, hyaluronic acid, alginate, and collagen.

2.4.2 For 4DPC

The materials used for 4DPC are commercially available polymer composite materials. These have been used to make structures in aircraft, automobiles, wind turbine blades, and so on. The mechanism for the change in shape is anisotropy. This is created by the different fiber orientations in different layers. An example is given below for illustration.

2.4.2.1 Illustrative example

Consider a laminate made with carbon/epoxy with a lay-up sequence of [0/90]. The thickness of each layer is 0.125 mm. The prepregs are laid down on a flat mandrel, with all ancillary materials contained in a vacuum bag. The vacuum bag is placed inside an autoclave for curing. The curing temperature is 177 °C. During curing, the resin hardens and becomes stiff. The soft laminate becomes a stiff laminate. At that temperature, the laminate is flat. The temperature is then cooled to room temperature. During the cooling process, there is interaction between the deformation of the 0° layer and the 90° layer. This interaction will create strains and curvatures. The strains and curvatures can be calculated using eqs. (1.88) and (1.89). For the curvatures, the equation is repeated here for clarity:

$$k_x^o = \frac{N_x^T B_{11} - M_x^T (A_{11} + A_{12})}{B_{11}^2 - (D_{11} - D_{12})(A_{11} + A_{12})} \quad (2.1)$$

The engineering constants are given in Table 1.1. Using these values, the stiffness components of the laminate can be calculated as follows:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} = \frac{155}{1 - 0.004} = 155.7 \text{ GPa} \quad (2.2)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} = \frac{12.1}{1 - 0.004} = 12.16 \text{ GPa} \quad (2.3)$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{(0.248)(12.1)}{1 - 0.004} = 3.02 \text{ GPa} \quad (2.4)$$

From eqs. (1.67) to (1.69)

$$A_{11} = h(Q_{11} + Q_{22}) = (0.125 \text{ mm})(155.7 + 12.16) \text{ GPa} = 20.98 \text{ MN/m} \quad (2.5)$$

$$A_{12} = 2hQ_{12} = 2(0.125 \text{ mm})(3.02 \text{ GPa}) = 0.755 \text{ MN/m} \quad (2.6)$$

$$B_{11} = \frac{h^2}{2}(Q_{22} - Q_{11}) = \frac{(0.125 \text{ mm})^2}{2}(12.16 - 155.7) \text{ GPa} = -1.12 \times 10^{-3} \text{ MN} \quad (2.7)$$

$$D_{11} = \frac{h^3}{3}(Q_{11} + Q_{22}) = \frac{(0.125 \text{ mm})^3}{3}(155.7 + 12.16) \text{ GPa} = 0.109 \times 10^{-6} \text{ MN m} \quad (2.8)$$

$$D_{12} = \frac{2h^3}{3}Q_{12} = \frac{2(0.125 \text{ mm})^3}{3}(3.02 \text{ GPa}) = 0.00393 \times 10^{-6} \text{ MN m} \quad (2.9)$$

From eq. (1.70)

$$\begin{aligned} N_x^T &= \Delta T h [(Q_{11} + Q_{12})\alpha_1 + (Q_{22} + Q_{12})\alpha_2] = (20 - 177)(0.125 \text{ mm})[(158.72 \text{ GPa}) \\ &\quad (-0.018 \times 10^{-6}) + (15.18 \text{ GPa})(24.3 \times 10^{-6})] = -7,295 \text{ N/m} \end{aligned} \quad (2.10)$$

From eq. (1.72)

$$\begin{aligned} M_x^T &= \frac{1}{2}h^2\Delta T [(Q_{12} - Q_{11})\alpha_1 + (Q_{22} - Q_{12})\alpha_2] = \frac{1}{2}(0.125 \text{ mm})^2(20 - 177) \\ &\quad [(3.02 - 155.7) \text{ GPa}(-0.018 \times 10^{-6}) + (12.16 - 3.02) \text{ GPa}(24.3 \times 10^{-6})] \\ &= -1.227(2.75 + 222.1) \times 10^{-3} = -0.276 \text{ N} \end{aligned} \quad (2.11)$$

Substituting into eq. (2.1) yields

$$\begin{aligned} k_x^o &= \frac{N_x^T B_{11} - M_x^T (A_{11} + A_{12})}{B_{11}^2 - (D_{11} - D_{12})(A_{11} + A_{12})} \\ &= \frac{(-7,295 \text{ N/m})(-1.12 \times 10^{-3} \text{ MN}) + (0.276 \text{ N})(21.74 \text{ MN/m})}{(-1.12 \times 10^{-3} \text{ MN})^2 - (0.105 \times 10^{-6} \text{ MN m})(21.74 \text{ MN/m})} = -13.77/\text{m} \end{aligned} \quad (2.12)$$

This gives the radius of curvature of

$$R_x = \frac{1}{k_x} = \frac{100 \text{ cm}}{13.77} = 7.26 \text{ cm} \quad (2.13)$$

Note that the curvature and radius of curvature depend on the value of layer thickness h . Sometimes an h value of 0.13 mm may be used, and this can give a different value of curvature.

Note that for the case of symmetric laminates, from the basic definitions in eqs. (1.28) and (1.29), it can be shown that:

$$B_{ij} = 0 \text{ and } M_x^T = 0$$

Then, $k_x = k_y = 0$ and the laminate will not be curved.

It can be seen in the above example that the curvature of the resulting structure is due to the interaction between the different layers caused by the difference in fiber orientations in these layers. The example only shows two layers of 0° and 90° , but similar calculations can be done for laminates containing different numbers of layers and at different fiber angles.

2.5 Conclusion

In this chapter, the similarities and differences between regular 4DP and 4DPCs have been presented. Even though both techniques use the term “4D printing,” the two techniques are very different from each other, from the materials used to the procedure for shape transformation to the final applications.

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Chapter 3

Procedure for 4D printing of composites

The method of 4D printing of composites (4DPC) consists of the following stages:

1. Determination of the feasibility of whether the part can be made using 4DPC
2. Determination of the lay-up sequence such that a flat stack of prepregs will curl up into the desired shape
3. Deposition of layers or strips of prepregs onto a flat mold
4. Bagging and curing the stack of layers
5. Use the structure as is after curing (U-structures), or make C-structures, or A-structures

3.1 Determination of the feasibility of whether the part can be made using 4DPC

The determination of whether a structure can be made by 4DPC depends on the shape of the structure and its intended use. Experiences over the past several years show that three types of structures can be made: U-structures, C-structures, and A-structures.

U-structures stand for unconstrained structures. These are structures with free edges. One example of the U-structure is shown in Figure 3.1. The structure has the shape of a flower. This flower has four petals, and each petal has five leaves. The leaves have free edges. The shape of these structures may change with the change in temperature or moisture absorption. The sensitivity of the shape to these environmental conditions will be discussed in Chapter 5. This change in shape due to environmental conditions can be utilized to make structures that are reactive to changes in environmental conditions.

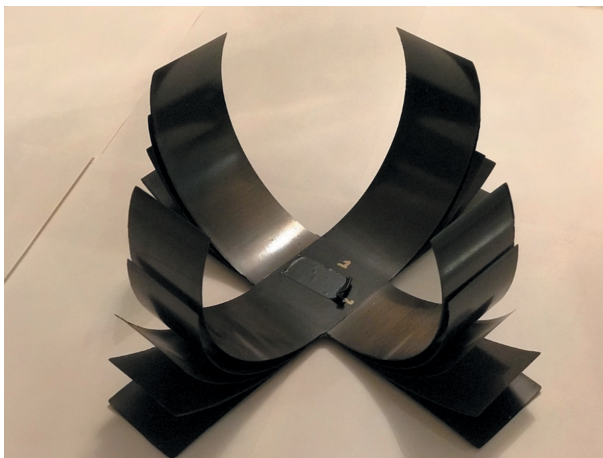


Figure 3.1: Composite flower: an example of a U-structure.

C-structures are those where the free edges are constrained. Figure 3.2 shows an example of a C-structure. This is the case of a stiffener having the shape of the omega symbol. This stiffener has a flat base and an upper part that is curved. The flat base and the upper part with bonding on the left side can be made in a single step using 4DPC. However, the right bonding area needs to be secondary bonded. As such, there is an extra step after the 4DPC process. Due to the fact that the free edges are constrained, the structure is not as sensitive to the change in environmental conditions (temperature and moisture) as compared to the U-structures.

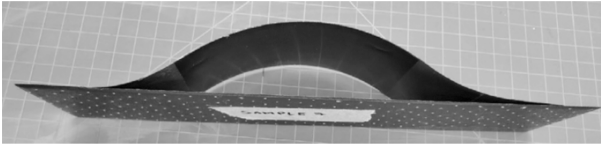


Figure 3.2: An example of a C-structure: an omega stiffener [1].

A-structures are C-structures with common shapes, such as A-cylinders or A-cones. The letter A stands for “almost.” They have the shape of the cylinder or cone, but there is a

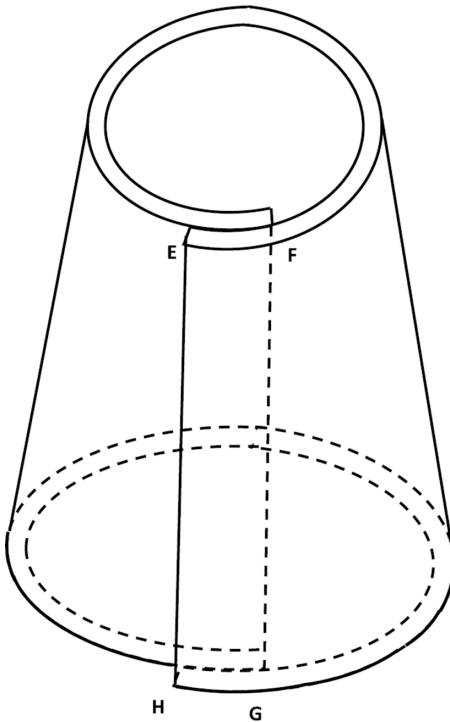


Figure 3.3: An example of an A-structure: an A-cone.

seam joining the edges together. Figure 3.3 shows an example of an A-cone. As such, they are not perfect cylinders or cones. Even though they are not perfect cylinders or cones, they can be made without the need for a mold, which can offer an alternative to the manufacturing process. It has also been demonstrated that their mechanical performance is equivalent to that of structures made using the conventional method, as will be presented in Chapter 7.

3.2 Determination of the lay-up sequence

The determination of the lay-up sequence can be done either in the forward route or in the backward route. In the forward route, the lay-up sequence can be proposed, and the configuration of the part can be derived either from calculation or from experimental tryout. This exploratory approach is suitable for discovering new shapes. In the backward approach, a shape of a part is given a priori. The lay-up sequence needs to be determined such that the 4DPC process will yield the part shape. Several examples presented in Chapter 5 utilize either one of these two approaches or both. A detailed presentation of the lay-up sequences can be found there.

3.3 Deposition of layers or strips of preregs on a flat mold

The deposition of layers of preregs is done using the hand lay-up (HLU) method. The deposition of strips is done using the automated fiber placement (AFP) machine.

3.3.1 Hand lay-up (HLU)

In HLU, a roll of prepreg (as shown in Figure 2.2) is taken out of the freezer and left at room temperature for thawing. These sheets of prepreg are cut into pieces with the dimensions of the desired part. The orientation of fibers in each piece would have been determined beforehand, depending on the loading conditions on the part. Each layer needs to be compacted against the previous layer. Compaction is essential to ensure a minimum amount of voids in the final part. In addition to Figure 2.3, Figure 3.4 shows another view of the HLU of the layer (the white part is the backing material, which will be peeled off after deposition) against a previous layer (black bottom layer). Hand pressing using a roller is the conventional way to compact a layer against a previous layer.

Figure 3.5a shows a stack of layers in a symmetric laminate, and Figure 3.5b shows a stack of layers in an unsymmetric laminate.



Figure 3.4: Placing the top layer on top of the bottom layer. When the backing paper (white) is peeled off the top layer, it will appear black, similar to the bottom layer.

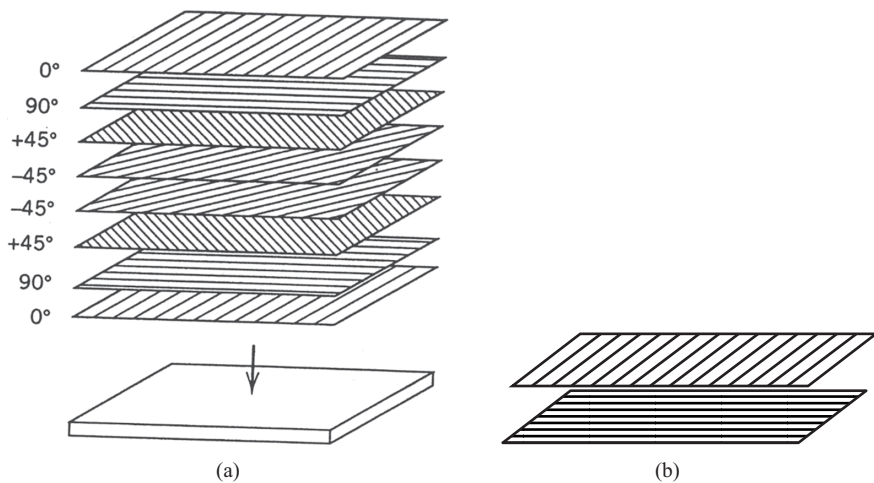


Figure 3.5: Stack of layers: (a) eight-layer symmetric laminate and (b) two-layer unsymmetric laminate.

3.3.2 Automated fiber placement (AFP)

In AFP, the pressing is done by a roller under a compaction force. Rather than a whole layer being laid down, in AFP, individual strips of narrow width (about 6.35 mm) are pressed down one at a time. Figure 3.6 shows a schematic of the AFP process. In this figure, the towpreg (a strip of the prepreg with a width of 6.35 mm or larger) is fed to the bottom of a roller (this location is called the NIP point). There is a heat source (ei-

ther a laser, hot gas torch, infrared, or heat lamp) that heats the tow prepreg to make it sticky to the layers that were laid before (called the substrate). There is a compaction system that pushes the roller downward. This creates compaction between the currently deposited prepreg and the substrate. Bonding is formed between the layer currently deposited and the layers in the substrate.

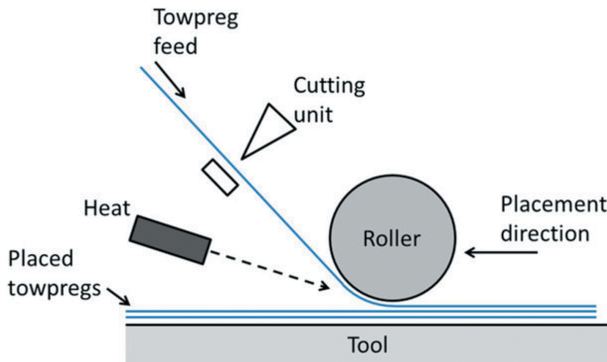


Figure 3.6: Schematic of the automated fiber placement process.

3.4 Bagging the stack of layers

The stack of layers needs to be bagged before it can be cured in either an oven or an autoclave. The purpose of the bagging is to help remove volatiles created during the heating of the resin. It also helps to impose vacuum on the laminate. Bagging consists of placing many ancillary materials on both faces of the stack of prepregs. Figure 3.7 shows a schematic of the different layers in the bagging process.

The bagging starts with a tool with a good surface finish. The tool can be made of aluminum or some other type of metal. The functions of the different materials are:

- Release agent is used to avoid the sticking of the laminate to the tool and to facilitate the removal of the part after curing.
- Teflon film serves as an additional release agent.
- Peel ply helps to remove the part after curing and provides some texture appearance for the surface.
- The bleeder is used to absorb excess resin that may flow from the stack of layers when the viscosity of the resin is low.
- The vacuum bag is used to maintain the vacuum in the laminate.

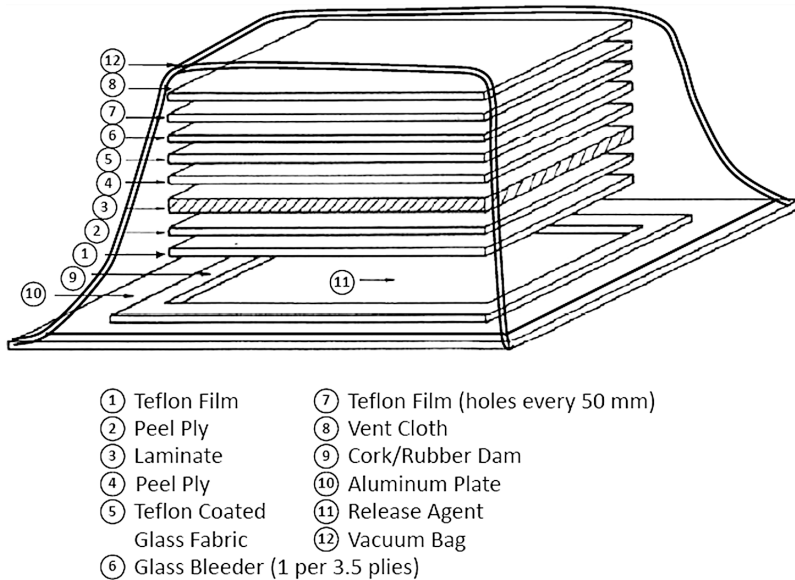


Figure 3.7: Layers in the bagging process [2].

3.5 Heating and curing the stack of layers

When the resin is first heated, its viscosity is reduced. This facilitates resin flow and wetting of the fibers. Subsequent heating increases the degree of cure of the resin. The schedule of heating (curing cycle) is important to ensure both wetting and proper cure within a reasonable amount of time. The schedule of heating depends on the type of resin system. It may be obtained from the resin or prepreg supplier, or from experience. More details can be found in [2]. An example of curing cycle is shown in Figure 3.8.

Figure 1.6 shows one example of the curing cycle for one type of carbon/epoxy composite material. The maximum cure temperature is 177 °C with a pressure of 0.31 MPa (45.6 psi). Figure 3.8 shows the curing cycle for another type of carbon/epoxy composite material. It recommends a maximum temperature of 275 °F (135 °C) and a maximum pressure of 40 psi. The curing cycle depends on the type of composite material and can be obtained from the material manufacturer.

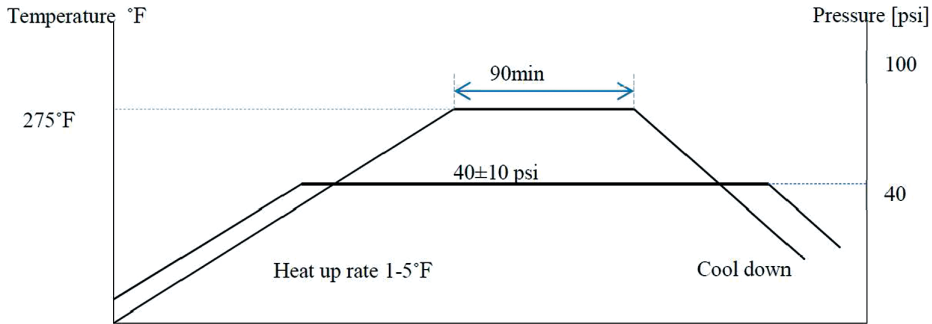


Figure 3.8: An example of curing cycle.

Heating can be done in an autoclave, an oven, a hot plate, or a heat blanket.

- **Autoclave:** The autoclave can provide both heat and pressure. The application of some pressure can result in better quality for the laminate, minimizing the amount of voids. For aircraft quality parts, the use of an autoclave is recommended. Figure 3.9 shows an autoclave at Concordia.



Figure 3.9: Autoclave at Concordia.

- **Oven:** The oven can only provide heating. Some pressure can be applied by the use of a vacuum. Due to the lack of high pressure, it is necessary to use resin or prepreg materials that have low viscosity. There are out-of-autoclave resins or prepreps suitable for this purpose. Note that the autoclave has a cylindrical

shape, while the oven has a box shape. The cylindrical shape is more efficient in containing pressure.

- **Hot plate:** Since only a flat mold is required, a hot plate can be an alternative for heating. Hot plates are simpler and less expensive than either an autoclave or an oven. In this case, a vacuum can also be applied to provide some pressure.
- **Heat blanket:** For the construction of large structures (dimensions in terms of meters), the autoclave, oven, or hot plate may not be practical. In this case, out-of-autoclave prepreg materials may be used, and the heat blanket can be an alternative form of heating. Vacuum can be applied to provide some pressure for the compaction.

3.5.1 Heating and curing multiple samples simultaneously

Due to the fact that a curved mold is not necessary, there is the possibility of laying and curing multiple pieces at the same time. A sheet made of nonstick material such as copper shim or Kapton film can be placed between the lay-ups for separate pieces.

3.6 Using the structure as is after curing

After being taken out of the heating chamber, the curved composite structure made by 4DPC can be used as is if it is the intended shape of the structure. The leaf spring (Figure 3.10) is such a structure. This is one example of the group of U-structures.



Figure 3.10: Composite leaf spring.

3.7 Constrained structures

The technique of 4DPC can provide a means to obtain the shape of the structures without the need for a complex mold. Fresh out of the autoclave or oven, the piece has free edges. For some applications, these edges can be constrained to conform to the final configuration. One example is the blade for a vertical axis wind turbine, as shown in Figure 3.11. The edges of the curved laminate are bolted to a fixed frame to conform to the desired configuration. Due to these constraints, the shape of these structures is not sensitive to environmental conditions.



Figure 3.11: Blade of a vertical axis wind turbine.

3.8 A-structures

To make the A-structures, two situations may exist. In the first situation, the length of the blank (flat composite prepregs) is not long enough such that after curing and cooling to room temperature, the edges of the final structure do not overlap. In order to make an A-structure out of the shell, the edges need to be bent further to create the overlap. In this case, it is necessary to ensure that the extra bending does not affect the curvature of the final A-structure. In the second situation, the length of the original blank is long enough (calculations can be done to ensure that sufficient length is used), and the edges of the structure fresh out of the autoclave are already overlapping. Adhesives can then be used to bond the edges together to make the A-structure. Figure 3.3 shows an example of an A-structure.

3.9 Conclusion

This chapter presented the procedure used in 4DPC to make different types of structures. For the unconstrained structures (U-structures), only the 4DPC procedure is required. For the constrained structures (C-structures) and A-structures, a secondary process is required.

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Chapter 4

Nonlinear aspects of the shape transformation of composite structures made by 4D printing of composites

The transformation from flat to curved configurations in the process of 4D printing of composites (4DPC) involves large deformation. Even though it was mentioned in Chapter 1 that the curvature obtained from laminate theory may be used to estimate the radius of curvature in the final structure made by 4DPC, the nonlinear aspect of the deformation needs to be taken into account if one were to determine the final shape of the structure using the radius of curvature calculated from the laminate theory. This is illustrated in this chapter. The illustration is done using the simplest type of laminate, but the procedure should be applicable to other types of laminates.

For laminates made of 0° and 90° layers, such as $[0_m/90_n]$, the equation for the out-of-plane displacement was given in eq. (1.39) as follows:

$$w^o = -\frac{1}{2} (K_x^o x^2 + K_y^o y^2) \quad (4.1)$$

At room temperature, for cases where only one curvature exists, the equation becomes

$$w^o = -\frac{1}{2} K_x^o x^2 \quad (4.2)$$

Differentiating eq. (4.2) twice gives the curvature a :

$$a = K_x^o \quad (4.3)$$

and the absolute value of the radius of curvature R is

$$R = \frac{1}{a} = \frac{1}{K_x^o} \quad (4.4)$$

From the specification of the lay-up sequence, the material properties, and the difference between the cure temperature and room temperature, one can obtain the curvature a . Using eq. (4.2), the out-of-plane displacement is given as follows:

$$w^o = -\frac{1}{2} a x^2 \quad (4.5)$$

If one were to plot eq. (4.5), a parabola as shown in Figure 4.1 would be obtained. This figure shows the shape of the laminate at different temperatures. At $\Delta T = 0^\circ\text{C}$ (cure temperature), the laminate has a flat shape. At $\Delta T = -30^\circ\text{C}$ (actual temperature equals 147°C), the laminate has the shape of a mild parabola. The radius of curvature changes continuously

as one moves away from the origin. Similar behavior is observed for other temperatures. As such, it can be seen that only a portion of the curves plotted from eq. (4.5) fits the shape of a circle. This is the same even at $\Delta T = -157^\circ\text{C}$ (actual temperature equals 20°C).

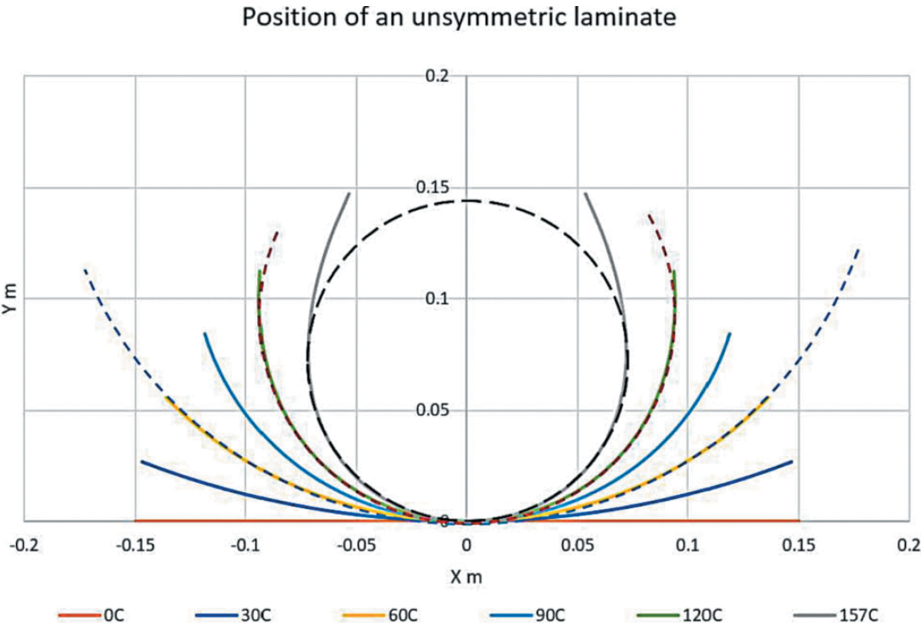


Figure 4.1: Plot of eq. (4.5). ΔT shows the difference between the stated temperature and the cure temperature.



Figure 4.2: Observed shape in reality.

The curves in Figure 4.1 do not agree with what is observed in reality, particularly at room temperature. What is observed in reality is a cylinder with a circular section, as shown in Figure 4.2.

In order to obtain the shape in Figure 4.2, eq. (4.5) needs to be plotted in a nonlinear fashion as shown below.

A new procedure is proposed below for the determination of the final shape of the laminate (the presentation below is extracted from reference [1]).

4.1 Proposed procedure

Assuming that the laminate has a single curvature (such as that of the $[0/90]$ laminate), the procedure to determine the shape is as shown in Figures 4.3 and 4.4 [1]. Let the center point of the laminate be O , the x -axis is along one in-plane direction, and the z -axis represents the out-of-plane direction. Assuming that the out-of-plane deformation is given by the equation:

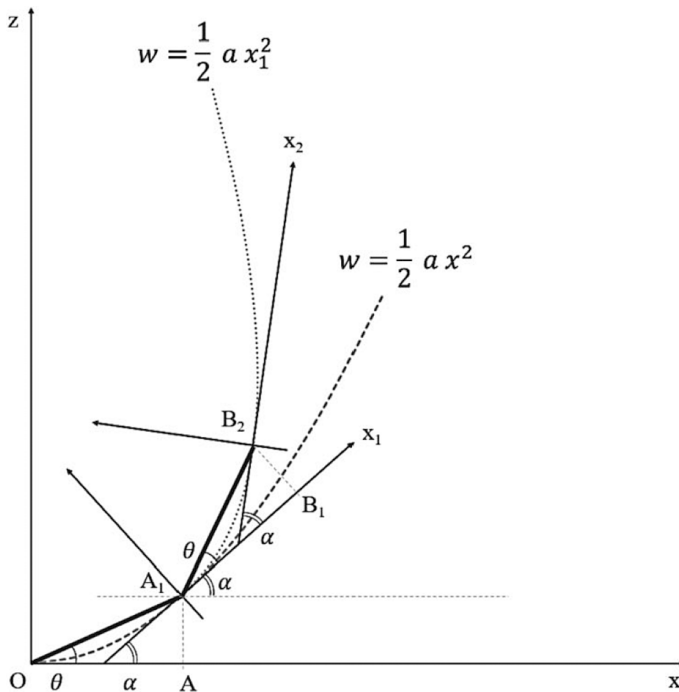


Figure 4.3: The first two steps of the procedure [1].

$$w = \frac{1}{2}ax^2 \quad (4.6)$$

where a is the curvature. Let the dimension of the laminate along the x -axis be divided into equal segments, each of length Δx . The endpoints of the intervals are denoted as A, B, C, D, E , and so on. The deformation of the laminate is determined increment-by-increment as follows.

4.1.1 First increment

As shown in Figure 4.3, point A will move along the z -direction by an amount equal to w_1 , which is determined by eq. (4.6) with $x = \Delta x$. The new position of point A is now A_1 . The coordinate axis is now moved from Ox to A_1x_1 .

The position of A_1 is Δx in the x -direction and $\frac{1}{2}a\Delta x^2$ in the z -direction. This position can be expressed as follows:

$$A_{1x} = R \cos \theta \quad (4.7)$$

$$A_{1z} = R \sin \theta \quad (4.8)$$

where R is the straight distance from O to A_1 , and θ is the angle between Ox and OA_1 :

$$R = \sqrt{(\Delta x)^2 + \left(\frac{1}{2}a\Delta x^2\right)^2} \quad (4.9)$$

$$\theta = \arctan\left(\frac{\frac{1}{2}a\Delta x^2}{\Delta x}\right) = \arctan\left(\frac{1}{2}a\Delta x\right) \quad (4.10)$$

The coordinate now is changed from Ox to A_1x_1 , where A_1x_1 is tangent to the curve $\widehat{OA_1}$ at A_1 . The angle between the new coordinate A_1x_1 and the original coordinate Ox is

$$\alpha = \arctan(w') = \arctan(a\Delta x) \quad (4.11)$$

where w' is the derivative of w with respect to x .

4.1.2 Second increment

The origin of the coordinate is now point A_1 . Relative to A_1 , point B_1 will move in the direction perpendicular to the axis A_1x_1 by an amount w_2 , which again is equal to $\frac{1}{2}a\Delta x^2$. Point B_1 now becomes B_2 , as can be observed in Figure 4.3.

The position of B_2 in the original coordinate system (Oxz) can be defined as the coordinates of A_1 plus the projection of A_1B_2 on the original coordinate, considering that the angle between A_1B_2 and Ox is $\theta + \alpha$:

$$B_{2x} = A_{1x} + R \cos(\theta + \alpha) = R \cos \theta + R \cos(\theta + \alpha) = R[\cos \theta + \cos(\theta + \alpha)] \quad (4.12)$$

$$B_{2z} = A_{1z} + R \sin(\theta + \alpha) = R \sin \theta + R \sin(\theta + \alpha) = R[\sin \theta + \sin(\theta + \alpha)] \quad (4.13)$$

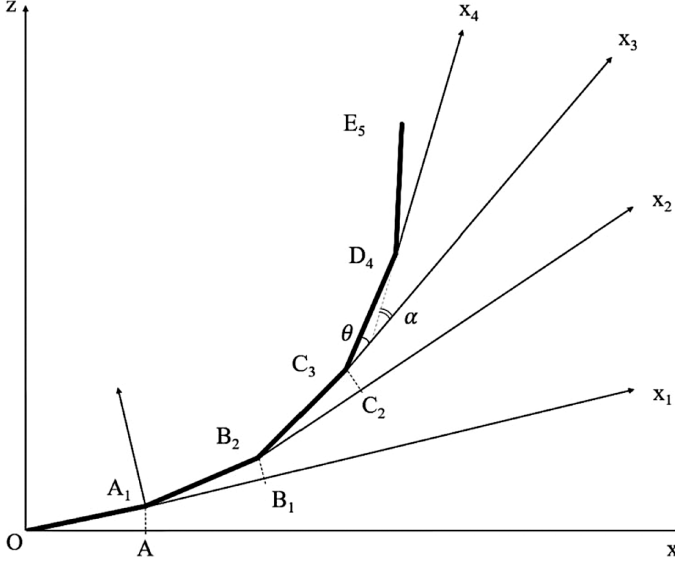


Figure 4.4: Many steps in the procedure.

4.1.3 Third increment

The origin of the coordinate now is point B_2 . Relative to B_2 , point C_2 will move in a direction perpendicular to the axis B_2x_2 by an amount w_3 , which again is equal to $\frac{1}{2}a\Delta x^2$. Point C_2 now becomes point C_3 .

The coordinates of C_3 can be calculated the same way as B_2 . They are the coordinates of B_2 plus the projection of B_2C_3 on the original coordinate, considering that the angle between B_2C_3 and Ox is $\theta + 2\alpha$:

$$\begin{aligned} C_{3x} &= B_{2x} + R \cos(\theta + 2\alpha) = R \cos \theta + R \cos(\theta + \alpha) + R \cos(\theta + 2\alpha) \\ &= R[\cos \theta + \cos(\theta + \alpha) + \cos(\theta + 2\alpha)] \end{aligned} \quad (4.14)$$

$$\begin{aligned} C_{3z} &= B_{2z} + R \sin(\theta + 2\alpha) = R \sin \theta + R \sin(\theta + \alpha) + R \sin(\theta + 2\alpha) \\ &= R[\sin \theta + \sin(\theta + \alpha) + \sin(\theta + 2\alpha)] \end{aligned} \quad (4.15)$$

This procedure can be repeated for other points D_4 , E_5 , and so on. Therefore, the general expression for the coordinates of the point P_n can be defined as

$$P_{nx} = R \sum_{i=1}^n \cos[\theta + (i-1)\alpha] \quad (4.16)$$

$$P_{nz} = R \sum_{i=1}^n \sin[\theta + (i-1)\alpha] \quad (4.17)$$

As shown in Figure 4.4, the final shape of the laminate follows the line OA_1, B_2C_3, D_4E_5 , and so on. By taking smaller and smaller increments of Δx , the shape converges to the final shape.

Using the mentioned method, the final displacement of an unsymmetric $[0/90]$ laminate is found and presented in Figure 4.5.

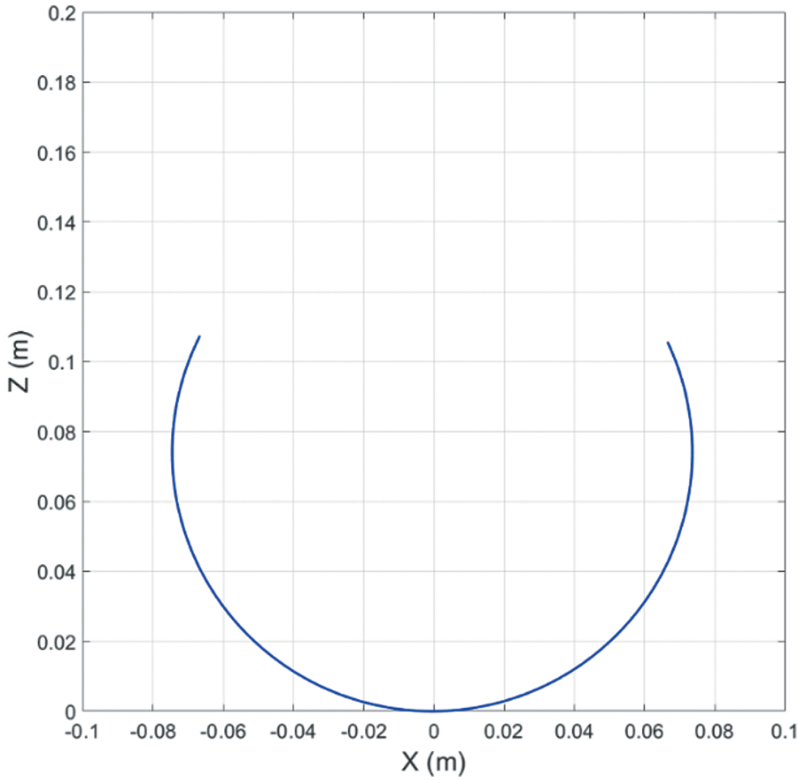


Figure 4.5: Room temperature shape of laminate $[0/90]$.

Reference

- [1] Suong V. Hoa and Emad Fakhimi, "Procedure to determine the deformed shape of laminates made by un-symmetric lay-up sequences: Basis for 4D printing of composites", *Composite Structures*, 295, 2022, 115748.



Part II: **Different structures**

Part II presents the 4DPC procedures that have been used to make a few composite structures such as U-structures (U stands for unconstrained), C-structures (C stands for constrained), and A-structures (A stands for “almost”). The U-structures refer to those that are made using the process of 4DPC only, and without secondary process. These structures have free edges. They may be affected by the fluctuation of the environmental conditions such as temperature and moisture. From the development so far, these include the S-shaped pieces, the composite flower, letters of the alphabet, the leaf springs, and the twisted pieces. The C-structures are those where the edges are constrained. The constraint can be done by bolting to a fixed frame, or by using an additional process such as bonding. Due to the constraint, the shape of these structures may not be affected by the change in the environmental conditions. These include the stiffeners and corrugated core for flexible aircraft wings. The A-structures are those having the shape of common objects such as cylinders or cones. However, due to the fact that the edges are bonded, rather than perfect body of revolution, these are called almost structures, such as A-cylinders and A-cones.

Chapter 5

U-structures

The following presents U-structures for which the manufacturing procedures using 4D printing of composites (4DPC) have been developed. The technology is ongoing. There are other structures that may be developed in the future.

5.1 S-shaped structures

A few S-shaped structures were made [1]. The S-shaped structure represents the curved structure that contains three features: cylindrical bending in two opposite directions, connections area, and edge effect. In order to provide cylindrical bending along two opposite directions, the flat lay-up needs to have two parts with lay-up sequences opposite to each other; for example, the left part with lay-up sequence $[0_m/90_n]$ and the right part with lay-up sequence $[90_n/0_m]$. These two parts need to be joined together to make a single structure. This can be done by an overlap area. The overlap area may have a symmetric lay-up and therefore may be straight. As such, this area should be kept at a minimum in order not to significantly affect the overall shape.

Figure 5.1 shows the flat lay-up form. The manufacturing steps are:

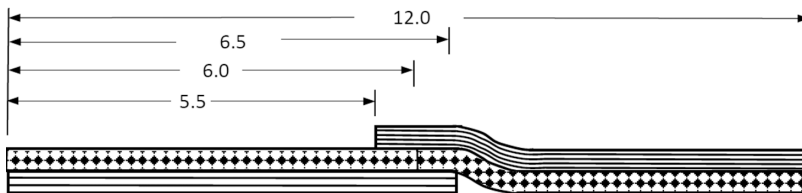


Figure 5.1: Flat lay-up for the S-shaped structure (dimensions in inches).

- A layer of 0° fibers is first placed on top of the flat mold, starting from the left end. The length of this layer is 6.5 in (165.1 mm).
- A second layer with fibers along the 90° direction is placed on top of the first layer. This layer has a length of 12.0 in (304.8 mm) and it runs along the whole length of the sample.
- A third layer with fibers along the 0° direction is placed on top of the second layer, starting from the right side. This layer has a length of 6.5 in (165.1 mm). It has an overlap length of 1.0 in (25.4 mm) with respect to the first layer.

This arrangement provides an unsymmetric laminate with a lay-up sequence of $[0/90]$ (starting from the bottom) on the left side, an unsymmetric laminate with a lay-up

sequence of $[90/0]$ on the right, and a symmetric laminate with a lay-up sequence of $[0/90/0]$ in the overlap area.

After curing and cooling to room temperature, a laminate with an S shape is obtained, as shown in Figure 5.2.



Figure 5.2: Final laminate with S shape.

A few observations are made:

- The S shape is the result of the unsymmetric laminates on the left and right sides.
- A trace of the shape of the piece is shown in Figure 5.3.
- The majority of the curve $ABCDE$ has a radius of curvature of 2.64 in (67 mm). This corresponds to a value of 2.14 in (54 mm) calculated using laminate theory. The discrepancy between the radius of curvature calculated from laminate theory and the experimental value was also observed by Hyer [2]. In Figures 1.9 and 1.10, it was observed that there were differences between the experimental value and the value calculated from laminate theory. The differences can be due to variations in the material properties and the microstructure of the material.
- At the edge region close to points A and E , the radius of curvature is larger (smaller curvature). This edge effect is due to the smaller amount of constraint caused by the lack of neighboring material.
- The overlap region BCD is flatter (larger radius of curvature) than the other region. This is due to the symmetric laminate in this region.
- The curvature of the final piece can be measured by one of two methods. The rough method is by tracing the curve and graphical measurement. The other method is to use the program Digimizer, where an image of the edge of the piece is taken. This image is analyzed using the program Digimizer [3].

5.2 Composite flower

Composite flower is a unique structure that can be made using only the method of 4DPC. None of the conventional methods of composite manufacturing can be used. A sample of the composite flower is shown in Figure 5.4. It can be seen that the flower has four petals, and each petal has five leaves. Each leaf has a different curvature.

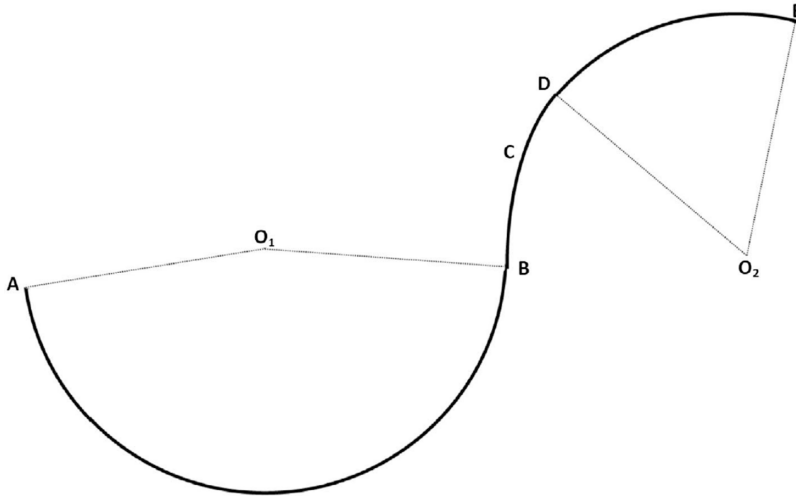


Figure 5.3: Trace of the edge of the S-shaped piece.

The conventional method of composite manufacturing utilizes a mold. It is not possible to have a mold with five different curvatures.

The lay-up procedure for this composite flower is shown in Figure 5.5. This figure shows the cross-section along two of the four petals. The manufacturing steps are as follows:

- A flat mold is prepared. The mold should have a length of at least 12 in (304.8 mm) to accommodate the length of the composite structure, plus room for attaching bagging materials.
- The surface of the mold should be polished to a smooth finish.
- A release agent is applied to the surface of the mold to avoid the sticking of the part to the mold and to facilitate part removal after curing.
- A stack of eight layers of prepregs is first deposited on the flat mold. The length of each layer is 7.5 in (190.5 mm) and its width is 1.5 in (38.1 mm). This stack has a symmetric lay-up sequence $[0_3/90_2/0_3]$. This will stay flat after curing and cooling to room temperature. It will serve as a base for the flower.
- A thin layer of brass shim is placed on top of the first stack of prepregs (shown in inclined hatched lines). This layer has a width of 1.5 in (38.1 mm). It has two parts. The left part is 3 in (76.2 mm) long and starts from the left end A and stops at B. The right part is 3 in (76.2 mm) long and starts from a point corresponding to C and ends at the right edge, corresponding to D. The function of the brass shim is to avoid sticking between the first layer of the second group of prepregs and the last layer of the first group of prepregs.
- The second stack of prepregs has a length of 8.0 in (203.2 mm) and a width of 1.5 in (38.1 mm). It extends 0.25 in (6.35 mm) beyond each end of the first stack of

layers. It has the unsymmetric lay-up sequence of $[0_4/90_4]$. This stack of layers will exhibit a slight curvature after curing and cooling.

- A thin layer of brass shim is placed on top of the second stack of prepregs. This layer of brass shim covers the whole length of the second stack of prepregs, except for the central portion corresponding to BC . There will be adhesion between the prepregs at the central section BC but no sticking over the portions corresponding to AB and CD .
- The third stack of prepregs has a length of 8.5 in (215.9 mm) and a width of 1.5 in (38.1 mm). It extends 0.25 in (6.35 mm) beyond each end of the second stack of prepregs. It has the unsymmetric lay-up sequence of $[0_3/90_3]$. This stack of layers will exhibit more curvature in comparison to the curvature of the second stack of layers.
- Another thin layer of brass shim is placed on top of the last layer of prepregs. This layer of brass shim has the same length as the layers in the third stack. It covers the whole length of the third stack of prepregs, except for the central portion corresponding to BC .
- The fourth stack of prepregs has a length of 9.0 in (228.6 mm) and a width of 1.5 in (38.1 mm). It extends 0.25 in (6.35 mm) beyond each end of the third stack of prepregs. It has the unsymmetric lay-up sequence of $[0_2/90_2]$. This stack of layers will exhibit more curvature in comparison to the curvature of the third stack of layers.
- Another thin layer of brass shim is placed on top of the last layer of prepregs. This layer of brass shim has the same length as the layers in the fourth stack. It covers the whole length of the fourth stack of prepregs, except for the central portion corresponding to BC .
- The fifth stack of prepregs has a length of 9.5 in (241.3 mm) and a width of 1.5 in (38.1 mm). It extends 0.25 in beyond each end of the fourth stack of prepregs. It has the unsymmetric lay-up sequence of $[0/90]$. This stack of layers will exhibit more curvature in comparison to the curvature of the fourth stack of layers.

The lay-up procedure for the direction perpendicular to the one above is similar. In order to avoid a pile-up in the central portion, the layers between the two directions should be alternated.

5.3 Leaf springs

Leaf springs have the curved shape as shown in Figure 5.6. Leaf springs are usually made out of metal and are used to support the weight of vehicles. They also serve the purpose of vibration control. Leaf springs used for automobiles usually consist of a few mono-leaves joined together. The thickness of a leaf spring varies from 3.2 to 16 mm. Its width varies from 32 to 125 mm. The height of a spring is about 1.5 in

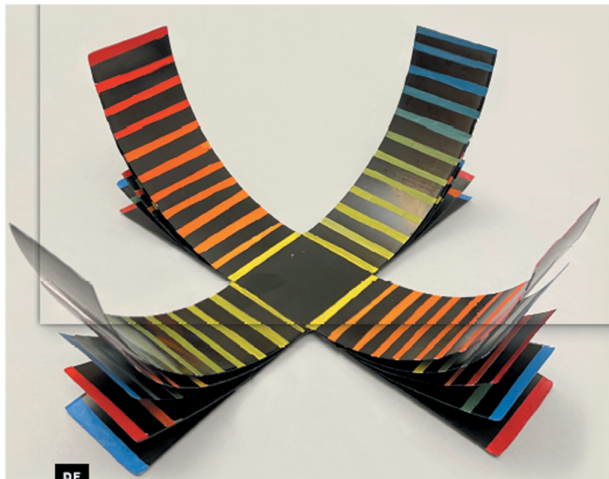


Figure 5.4: Composite flowers. Note that the colors were added to sharpen the image. The colors were not produced by 4DPC.

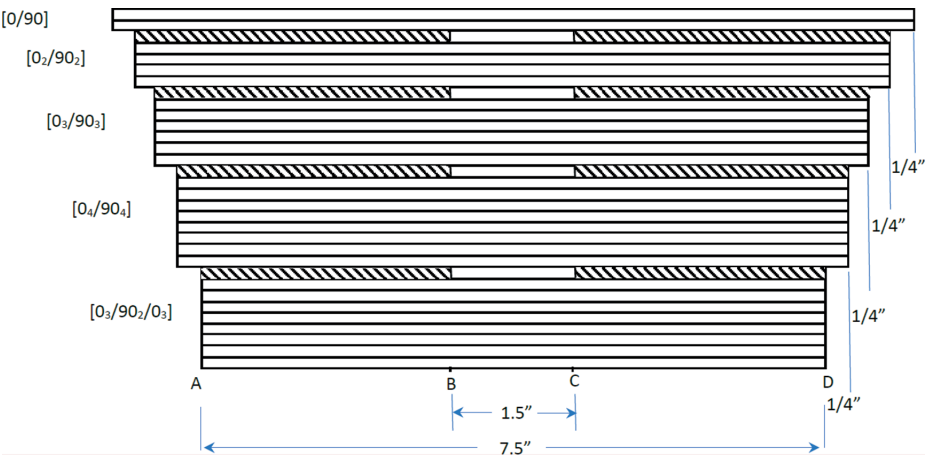


Figure 5.5: Lay-up procedure for the composite flower along one direction.

(38.1 mm). The ratio of load over displacement (spring constant) P/δ is about 800 lb/in or 1,400 N/cm. If two mono-leaves are joined together to make the spring, the spring constant for each mono-leaf is about 175 N/cm.

Table 5.1 provides a summary of the characteristics of a typical leaf spring. From the specifications of the length and height, the radius of curvature of the spring is about 236 in (6,000 mm). These values are used as guidelines for the development of composite leaf springs made by the 4DPC method.

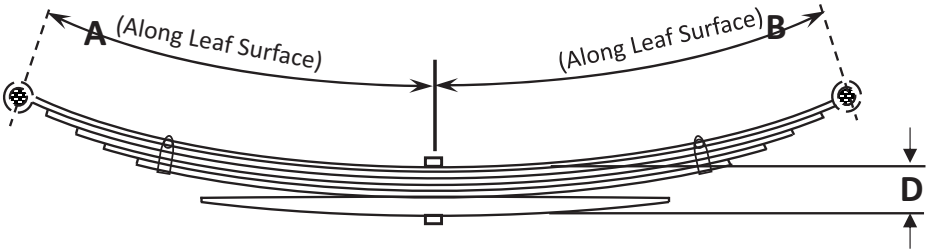


Figure 5.6: Configuration of a general leaf spring.

Table 5.1: Characteristics of a typical leaf spring.

Items	Specifications
Total length (mm)	1,000
Width (mm)	50
Arc height (mm)	40
Radius of curvature (mm)	6,000
Thickness of leaf (mm)	10
EI (steel $E = 205$ GPa)	854 N m^2
Spring constant (N/cm)	200

From the stiffness point of view, a single leaf spring can be idealized to take the shape of a beam, as shown in Figure 5.7. The load is applied at the mid-length of the beam. As such, the spring can be considered as two cantilever beams joined together at the center.

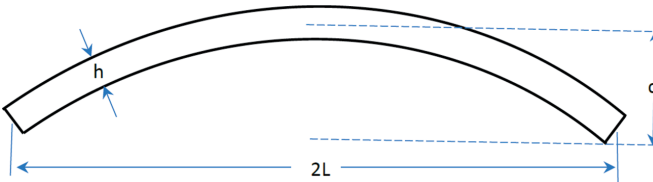


Figure 5.7: Idealized shape of leaf spring.

The stiffness of a mono-leaf can be estimated by the following equation for a cantilever beam:

$$\frac{P}{\delta} = \frac{1}{4} \frac{Ebh^3}{L^3} \quad (5.1)$$

where P is the applied load, δ is the displacement under the load point, P/δ is the stiffness constant, E is the modulus of the material (for isotropic material), b is the width of the beam, h is its thickness, and L is half the length of the leaf spring.

For example, a mono-leaf made of steel ($E = 205$ GPa), with a width of 50 mm, a thickness of $\frac{1}{4}$ in (6.35 mm), and a length ($2L$) of 24 in (609.6 mm), eq. (5.1) gives a stiffness constant:

$$\frac{P}{\delta} = \frac{1}{4} \frac{\left(\frac{205 \times 10^9 \text{ N}}{\text{m}^2} \right) (50 \times 10^{-3} \text{ m}) (6.35 \times 10^{-3} \text{ m})^3}{(0.3048 \text{ m})^3} = \frac{23,169 \text{ N}}{\text{m}} = 231.7 \text{ N/cm} \quad (5.2)$$

This value is used to estimate the characteristics of the steel spring, as given in Table 5.1.

The modulus E_1 of carbon/epoxy, as given in Table 1.1, is 155 GPa. As such, a carbon/epoxy leaf spring made of the material whose properties are given in Table 1.1 would have a thickness of $(205/155)^{1/3} = 1.097$ times the thickness of the steel spring in order to have an equivalent spring constant. However, with the specific gravity of steel being 7.8 and that of carbon/epoxy being 1.2, the weight of the composite spring would be $(1.097)(1.2/7.8) = 0.22$ times the weight of a steel leaf spring with similar length and width.

Work on composite leaf springs has been reported before [4]. In these works, a curved mold is made. Layers of either prepregs or fibers wetted with resin are applied on top of the curved mold. The material is then cured and cooled.

It would be of interest to see if the technique of 4DPCs can be used to make curved leaf springs without the need for a curved mold. One may argue that the cost of the mold may not be significant when there are a large number of springs to be made, and the benefit of 4DPC may not be significant. This is true for springs of small sizes and for large volume production. For springs of large sizes, custom-made springs, or situations where making a mold may not be possible or desirable, 4DPC provides a cost-effective and time-efficient alternative. In addition, since only flat tools are required, multiple springs may be made simultaneously by placing the lay-up of one sample over another, separated by some nonstick material.

5.3.1 Procedure to develop the composite leaf spring using 4DPC

The procedure consists of the following steps:

- Use Strength of materials to determine the stiffness constant of the composite spring as a function of the lay-up sequence.
- Selecting the configuration that provides the desired dimensions and stiffness constant.
- Manufacturing the selected samples.
- Test the samples.
- Evaluate the performance and compare it with specific values.

Use strength of materials to determine the characteristics of the composite leaf spring

The ratio between load and deformation at the mid-length of a curved beam, as shown in Figure 5.8, is given as [5]

$$\frac{P}{\delta} = \frac{EI}{R^3 \left(\theta_o - \sin \theta_o - \frac{\theta_o^2}{2} \sin \theta_o \right)} \quad (5.3)$$

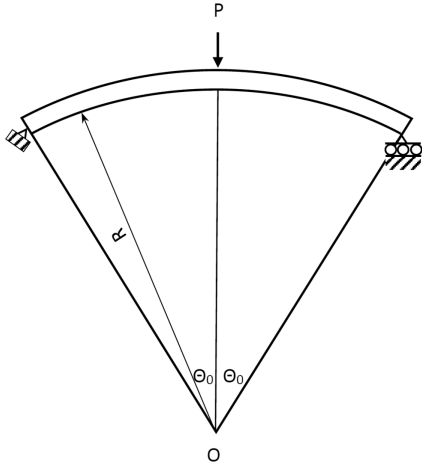


Figure 5.8: A curved beam loaded by a load at mid-length [5].

where δ is the displacement under the load, R is the radius of curvature of the beam, and θ_o is half the included angle of the beam. The curved length of the beam can be obtained from $G = 2R\theta_o$.

For the case of a spring made using 4DPC, R is created by the unsymmetry of the lay-up sequence in the beam. R can be obtained using laminate theory on the unsymmetric laminate. The included angle $2\theta_o$ can be obtained from the length G of the flat lay-up ($2\theta_o = G/R$).

EI is the flexural stiffness of the beam for isotropic materials. For beams made of composite materials, the equivalent flexural stiffness can be obtained by

$$\langle EI \rangle = (EI)_1 + (EI)_2 + \cdots + (EI)_n \quad (5.4)$$

where E is the Young's modulus of the material in a particular layer along the length direction of the beam, and I is the cross-sectional inertia of that layer. The subscripts 1, 2, . . . , n refer to the layer number in the beam.

Note that eq. (5.4) only holds for situations where fiber angles are either 0° or 90° . For other fiber angles, more involved equations are necessary.

5.3.2 Elastic axis

It is necessary to determine the flexural stiffness about the elastic axis of the section of the beam. Consider a beam made of m 0° layers and n 90° layers, as shown in Figure 5.9.

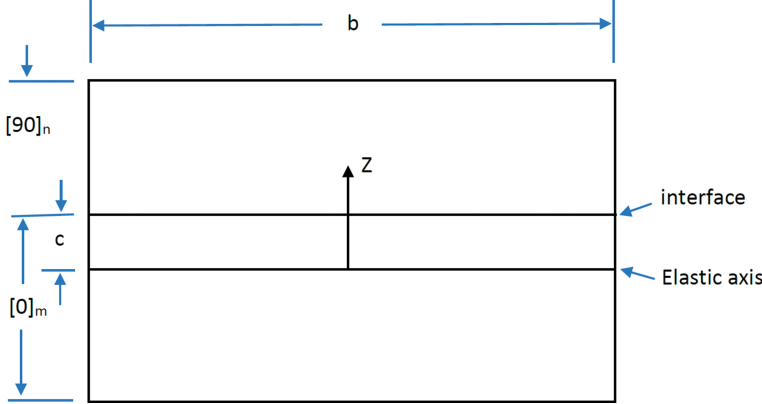


Figure 5.9: Interface and elastic axis.

The lay-up sequence can be written as $[0_m/90_n]$. The location of the elastic axis is obtained using the following equation:

$$\int E_i z \, dS = 0 \quad (5.5)$$

where E_i is the elastic modulus of layer i , and S is the cross-sectional area. Let b represent the width of the laminate, and c be the distance between the interface and the elastic axis. The above equation can be written as follows:

$$\int_{c-mh}^c b E_1 z \, dz + \int_c^{c+nh} b E_2 z \, dz = 0 \quad (5.6)$$

where h is the thickness of a single ply.

Integrating and simplifying give

$$c = \frac{h E_1 m^2 - E_2 n^2}{2 E_1 m + E_2 n} \quad (5.7)$$

Let the degree of anisotropy (E_1/E_2) be denoted as a :

$$a = \frac{E_1}{E_2} \quad (5.8)$$

Equation (5.7) can be written as

$$c = \frac{h}{2} \frac{am^2 - n^2}{am + n} \quad (5.9)$$

For the case of $m = n$

$$c = \frac{h}{2} \frac{m(a-1)}{a+1} \quad (5.10)$$

which can be written as

$$\frac{c}{mh} = \frac{1}{2} \frac{a-1}{a+1} \quad (5.11)$$

This indicates that for the case $m = n$, and for a fixed anisotropy ratio, the distance between the interface and the elastic axis is proportional to the thickness of the laminate.

For the case of $m = n = 1$

$$c = \frac{h}{2} \frac{a-1}{a+1} \quad (5.12)$$

A plot of c from eq. (5.12) versus a is shown in Figure 5.10.

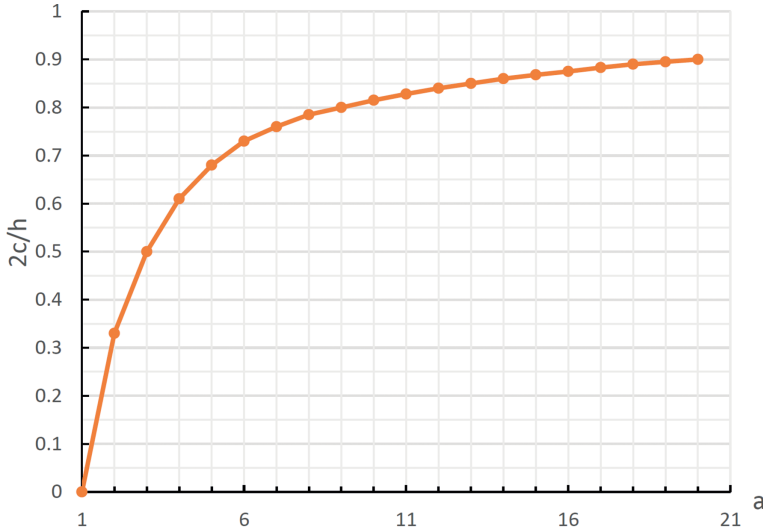


Figure 5.10: Plot of c versus the ratio $a = E_1/E_2$ for the case of a [0/90] laminate.

It can be seen that, in the case where two layers are made of the same material ($E_1 = E_2$), the elastic axis coincides with the interface. When the degree of anisotropy increases (a becomes larger and larger), the elastic axis moves away from the interface and goes more and more into the stiffer phase.

5.3.3 Equivalent flexural stiffness

Normally, the flexural stiffness of a beam is evaluated using the mid-plane as the axis. While this is accurate for the case of isotropic materials and symmetric laminates, for the case of unsymmetric laminates, the use of the mid-plane as the axis for evaluation of flexural stiffness may not be representative of the structural behavior. The elastic axis represents a location of zero strain, and the evaluation of the flexural stiffness about this axis may be more representative of the flexural behavior.

The equivalent flexural stiffness $\langle EI \rangle$ about the elastic axis is expressed as

$$\langle EI \rangle = \int E_i z^2 dz = \int_{c-mh}^c b E_1 z^2 dz + \int_c^{c+nh} b E_2 z^2 dz \quad (5.13)$$

where b is the width of the laminate.

This gives

$$\langle EI \rangle = \frac{bh}{3} \{ E_1 (3c^2 m - 3cm^2 h + m^3 h^2) + E_2 (3c^2 n + 3cn^2 h + n^3 h^2) \} \quad (5.14)$$

For the case $m = n$,

$$\langle EI \rangle = \frac{bhm}{3} \{ E_1 (3c^2 - 3cmh + m^2 h^2) + E_2 (3c^2 + 3cnh + n^2 h^2) \} \quad (5.15)$$

For the case $m = n = 1$,

$$\langle EI \rangle = \frac{bh}{3} \{ E_1 (3c^2 - 3ch + h^2) + E_2 (3c^2 + 3ch + h^2) \} \quad (5.16)$$

Using eq. (5.14), values of the equivalent flexural stiffness for a number of laminates with different stacking sequences can be obtained. These are given in Table 5.2 (material properties are from Table 1.1).

5.3.4 Effect of different stacking sequences on the radius of curvature using laminate theory: selection of lay-up sequence for leaf spring

It was shown previously that a laminate with a lay-up sequence of [0/90] would curl up into the shape of a cylindrical shell after curing and cooling to room temperature. Provided that the dimensions of this laminate are outside the range where bifurcation may occur, the radius of curvature of this laminate may be approximated using laminate theory. Laminates such as [0/90] contain only two layers. With a layer thickness of only 0.125 mm, two layers give a total thickness of 0.25 mm. This is too thin, and the equivalent EI for this laminate is too small to meet the stiffness requirements as shown in Table 5.1.

Table 5.2: Values of equivalent flexural stiffness $\langle EI \rangle$ for different laminates using properties from Table 1.1 ($b = 50$ mm, $h = 0.125$ mm, $E_1/E_2 = 155/12.1 = 12.8$).

Lay-up	No. of layers	Thickness (mm)	Elastic center c (mm), eq (5.7)	$\langle EI \rangle$ N m ² , eq. (5.14)	Calculated radius of curvature (cm) (from laminate theory)	Experimental radius of curvature (cm)
[0/90]	2	0.250	0.43 h	0.011	6.3	5.6–7.2
[0/90 ₂]	3	0.375	0.30 h		6.1	5.6–6.2
[0/90 ₃]	4	0.500	0.12 h		7.6	6.3–7.0
[0/90 ₄]	5	0.625	–0.10 h		9.5	
[0/90 ₅]	6	0.750	–0.34 h		11.5	
[0/90 ₆]	7	0.875	–0.62 h		13.7	
[0/90 ₇]	8	1.000	–0.91 h		15.9	14.6–15.9
[0/90 ₈]	9	1.125	–1.23 h		18.4	
[0/90 ₉]	10	1.250	–1.56 h		20.8	
[0/90 ₁₀]	11	1.375	–1.91 h		23.4	
[0 ₂ /90]	3	0.375	0.94 h		20.1	
[0 ₂ /90 ₂]	4	0.500	0.83 h	0.043	12.5	13.3
[0 ₂ /90 ₃]	5	0.625	0.74 h		11.6	13.8
[0 ₂ /90 ₄]	6	0.750	0.59 h		12.3	
[0 ₂ /90 ₅]	7	0.875	0.43 h		13.6	
[0 ₂ /90 ₆]	8	1.000	0.25 h		15.2	
[0 ₂ /90 ₇]	9	1.125	0.03 h		17.0	
[0 ₂ /90 ₈]	10	1.250	–0.19 h		18.9	
[0 ₂ /90 ₉]	11	1.375	–0.43 h		21.0	
[0 ₂ /90 ₁₀]	12	1.500	–0.67 h		23.1	
[0 ₈ /90 ₁₂]	20	2.500	2.95 h		46.4	60
[0 ₁₆ /90 ₂₄]	40	5.000	5.90 h	16.68	93	105
[0 ₂₄ /90 ₃₆]	60	7.500	8.88 h		139	

Consider the case of a laminate with a lay-up sequence $[0_{16}/90_{24}]$, $\langle EI \rangle = 16.68$ N.m². This laminate has a thickness of 40 layers or 5 mm. If comparison is to be made with a steel spring as shown in Table 5.1 with a thickness of 10 mm, the $\langle EI \rangle$ for the composite spring should be multiplied by $2^3 = 8$ to give 133.4 N cm². The value of $\langle EI \rangle$ for the steel spring given in Table 5.1 is 854 N m². Taking into account the ratio of the specific gravity of steel and composite to be 6.5 (7.8/1.2), the equivalent value of $\langle EI \rangle$ for the composite spring (with the same weight as the steel spring) would be $133.4 \times 6.5 = 867$ N.m². As such, the composite spring made using 4DPC would have similar flexural stiffness as the steel spring with equivalent weight.

A few springs with a laminate sequence of $[0_{16}/90_{24}]$ were made using carbon/epoxy. After curing and cooling to room temperature, curved beams were obtained. Figure 5.11 shows a typical curved beam. This beam has a thickness of 5.0 mm (40 layers at 0.125 mm each), a width of 3 in (76.2 mm), and a span length of 24 in (60.96 cm) (from an initial length of 24.2 in or 61.4 cm). The height from the plane of support to the lower edge of the beam is 1.25 in (3.175 cm).



Figure 5.11: Photo of a curved beam.

It is of interest to determine the mechanical characteristics of the beam and compare them to those of metallic beams of equivalent weight. The composite beam was subjected to three-point bending in an MTS (Materials Test Systems) testing machine, as shown in Figure 5.12.



Figure 5.12: Curved testing in an MTS machine.

Figure 5.13a shows the load–displacement curves for two beams of length 12 in (304.8 mm) and 24 in (609.6 mm), respectively. The static tests give a stiffness of 64.5 N/cm for the 24-in-long beam (width 76.2 mm and thickness 5.0 mm). This is in contrast with a value of 231.7 N/cm for a mono-leaf spring made out of steel (thickness = 6.35 mm, width = 50 mm, length = 609.6 mm) as given in eq. (5.2). If the steel spring had similar thickness, width, and length as the composite spring, its stiffness using eq. (5.1) would be 172.4 N/cm. The steel spring would have a spring constant that is 2.67 times more than the composite spring made by 4DPC. If weight consideration is

taken into account, with the specific gravity of composites being 1.5 and that of steel being 7.8 (5.2 times different), the composite springs would be attractive.

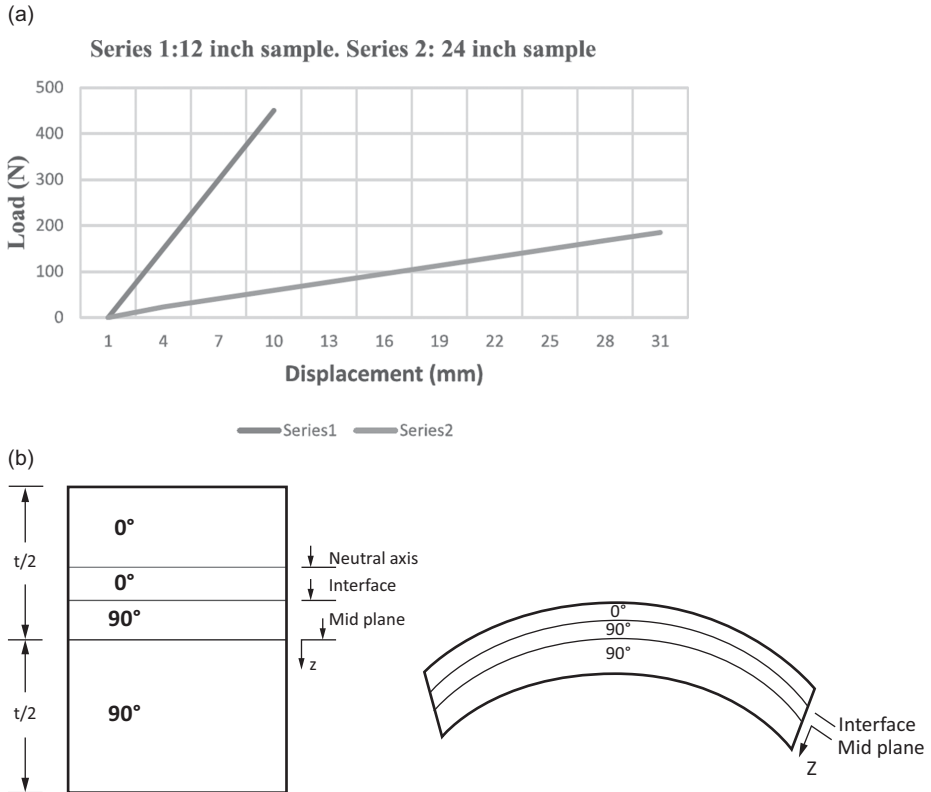


Figure 5.13: (a) Load-displacement curves for two beams under static tests [5]; (b) Locations of mid-plane, neutral surface, and interface (after manufacturing).

The springs were subjected to fatigue loading from completely curved to completely flat for more than 1 million cycles. They did not show any signs of degradation [5].

5.3.5 Explanation for the behavior of the composite spring made by 4DPC

The behavior of the curved beam made by 4DPC in three-point bending and in fatigue loading seems to contradict conventional expectations. The beam is concave downward with the 0° layers on top and the 90° layers at the bottom. When the curved beam is subjected to bending, as shown in Figure 5.12, the 90° layers are subjected to tensile strain while the 0° layers are subjected to compressive strain. Normally, the 90° layers have low tensile strength and tend to break easily. However, experimental

results show that the samples survived 1 million cycles of loading from curved to flat (and vice versa) without degradation. The reason is that the 90° layers experience compressive strain as the laminate is cooled from cure temperature to room temperature. The magnitude of the compressive strain due to manufacturing is larger than the magnitude of the tensile strain created by the bending load during mechanical loading operation. A detailed explanation is given below.

Under mechanical loading, the configuration of the composite spring is similar to that of Figure 5.8. For a lay-up sequence such as $[0_{16}/90_{24}]$, the upper layers are the 0° layers and the lower layers are the 90° layers. For durability, it is necessary that the strains developed during loading do not exceed the maximum strain that the material can withstand.

The curved laminate, as shown in Figure 5.8, was subjected to residual stresses due to the difference in coefficients of thermal contraction in the different layers during cooling from the cure temperature. Analysis can be done to determine these residual stresses. Analysis can also be done to determine the stresses due to mechanical loading, as shown in Figure 5.8.

5.3.5.1 Residual strains due to the cooling from the cure temperature to room temperature

For a laminate subjected to a temperature gradient of ΔT , the relation between the stress/moment resultants and strains and curvatures is

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (5.17)$$

For the case of cross-ply laminates, it can be shown that $A_{16} = A_{26} = B_{12} = B_{16} = B_{26} = D_{16} = D_{26} = 0$. Equation (5.17) can be shown to consist of two decoupled equations, one containing the x and y components and the other containing the xy component. For the x and y components, we have

$$\begin{bmatrix} N_x^T \\ N_y^T \\ M_x^T \\ M_y^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & 0 \\ A_{12} & A_{22} & 0 & B_{22} \\ B_{11} & 0 & D_{11} & D_{12} \\ 0 & B_{22} & D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} \quad (5.18)$$

Inverting yields

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_{11} & b_{12} \\ a_{12} & a_{22} & b_{21} & b_{22} \\ b_{11} & b_{12} & d_{11} & d_{12} \\ b_{21} & b_{22} & d_{12} & d_{22} \end{bmatrix} \begin{bmatrix} N_x^T \\ N_y^T \\ M_x^T \\ M_y^T \end{bmatrix} \quad (5.19)$$

The stresses at any particular point are given as follows:

$$\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ \overline{Q}_{12} & \overline{Q}_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{oT} + z\kappa_x^{oT} - \alpha_x \Delta T \\ \varepsilon_y^{oT} + z\kappa_y^{oT} - \alpha_y \Delta T \end{bmatrix} \quad (5.20)$$

where

$$\alpha_x = \alpha_1 m^2 + \alpha_2 n^2 \quad \alpha_y = \alpha_1 n^2 + \alpha_2 m^2 \quad (5.21)$$

$$N_x^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{11} \alpha_x^T + \overline{Q}_{12} \alpha_y^T + \overline{Q}_{16} \alpha_{xy}^T \right) \Delta T dz \quad (5.22a)$$

$$N_y^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{12} \alpha_x^T + \overline{Q}_{22} \alpha_y^T + \overline{Q}_{26} \alpha_{xy}^T \right) \Delta T dz \quad (5.22b)$$

$$M_x^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{11} \alpha_x^T + \overline{Q}_{12} \alpha_y^T + \overline{Q}_{16} \alpha_{xy}^T \right) \Delta T z dz \quad (5.22c)$$

$$M_y^T = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left(\overline{Q}_{12} \alpha_x^T + \overline{Q}_{22} \alpha_y^T + \overline{Q}_{26} \alpha_{xy}^T \right) \Delta T z dz \quad (5.22d)$$

5.3.5.2 Strains and stresses due to mechanical load

Since the only mechanical load is the bending moment M_x caused by the load P , the stress resultant–strain relation can be written as

$$\begin{bmatrix} 0 \\ 0 \\ M_x \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & 0 \\ A_{12} & A_{22} & 0 & B_{22} \\ B_{11} & 0 & D_{11} & D_{12} \\ 0 & B_{22} & D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} \quad (5.23)$$

Inverting yields

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_{11} & b_{12} \\ a_{12} & a_{22} & b_{21} & b_{22} \\ b_{11} & b_{12} & d_{11} & d_{12} \\ b_{21} & b_{22} & d_{12} & d_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ M_x \\ 0 \end{bmatrix} \quad (5.24)$$

Based on Kirchhoff's assumption, the strain at any point across the thickness of the laminate can be written as follows:

$$\varepsilon_x = \varepsilon_x^o + z\kappa_x = (b_{11} + d_{11}z)M_x \quad (5.25)$$

The neutral axis is located at

$$z = -\frac{b_{11}}{d_{11}} \quad (5.26)$$

The stresses due to mechanical load at any point can be given as

$$\begin{bmatrix} \sigma_x^M \\ \sigma_y^M \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ \overline{Q}_{12} & \overline{Q}_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{oM} + z\kappa_x^{oM} \\ \varepsilon_y^{oM} + z\kappa_y^{oM} \end{bmatrix} \quad (5.27)$$

5.3.5.3 Strains due to the combination of residual stresses and mechanical loads

The stresses due to the combination of thermal residual stresses and mechanical loads can be obtained using both eqs. (5.20) and (5.27):

$$\begin{bmatrix} \sigma_x^C \\ \sigma_y^C \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ \overline{Q}_{12} & \overline{Q}_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{oT} + z\kappa_x^{oT} - \alpha_x\Delta T + \varepsilon_x^{oM} + z\kappa_x^{oM} \\ \varepsilon_y^{oT} + z\kappa_y^{oT} - \alpha_y\Delta T + \varepsilon_y^{oM} + z\kappa_y^{oM} \end{bmatrix} \quad (5.28)$$

Example: The particular case of the laminate $[0_{16}/90_{24}]$ will be examined here. For this, using the properties in Table 1.1, one has

$$Q_{11} = 155.7 \text{ GPa} \quad Q_{12} = 3.02 \text{ GPa} \quad Q_{22} = 12.16 \text{ GPa} \quad Q_{66} = 4.40 \text{ GPa}$$

For 0° layer:

$$\begin{aligned} \overline{Q}_{11} &= 155.7 \text{ GPa} & \overline{Q}_{12} &= 3.02 \text{ GPa} & \overline{Q}_{22} &= 12.16 \text{ GPa} & \overline{Q}_{66} &= 4.40 \text{ GPa} \\ \overline{Q}_{16} &= 0 \text{ GPa} & \overline{Q}_{26} &= 0 \text{ GPa} \end{aligned}$$

For the 90° layer:

$$\begin{aligned} \overline{Q}_{11} &= 12.16 \text{ GPa} & \overline{Q}_{12} &= 3.02 \text{ GPa} & \overline{Q}_{22} &= 155.7 \text{ GPa} & \overline{Q}_{66} &= 4.40 \text{ GPa} \\ \overline{Q}_{16} &= 0 \text{ GPa} & \overline{Q}_{26} &= 0 \text{ GPa} \end{aligned}$$

Then,

$$\begin{aligned}
 A_{11} &= 3.48 \times 10^8 \text{ N/m} \\
 A_{12} &= 0.15 \times 10^8 \text{ N/m} \\
 A_{22} &= 4.91 \times 10^8 \text{ N/m} \\
 B_{11} &= 4.31 \times 10^5 \text{ N} \\
 B_{22} &= -B_{11} = 4.31 \times 10^5 \text{ N} \\
 D_{11} &= 868 \text{ Nm} \\
 D_{22} &= 880 \text{ Nm} \\
 D_{12} &= 31.4 \text{ Nm}
 \end{aligned}$$

Strains due to thermal (manufacturing) loads

Equation (5.18) becomes

$$\begin{bmatrix} N_x^T \\ N_y^T \\ M_x^T \\ M_y^T \end{bmatrix} = \begin{bmatrix} 3.48 \times 10^8 & 0.15 \times 10^8 & -4.31 \times 10^5 & 0 \\ 0.15 \times 10^8 & 4.91 \times 10^8 & 0 & 4.31 \times 10^5 \\ -4.31 \times 10^5 & 0 & 868 & 31.4 \\ 0 & 4.31 \times 10^5 & 31.4 & 880 \end{bmatrix} \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} \quad (5.29)$$

Using Mathematica to invert, one has

$$\begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} = 10^{-3} \begin{bmatrix} 7.5 \times 10^{-6} & -1.97 \times 10^{-7} & 3.7 \times 10^{-3} & -3.63 \times 10^{-5} \\ -1.97 \times 10^{-7} & 3.6 \times 10^{-6} & -3.42 \times 10^{-5} & -1.76 \times 10^{-3} \\ 3.7 \times 10^{-3} & -3.42 \times 10^{-5} & 3.01 & -9.0 \times 10^{-2} \\ -3.63 \times 10^{-5} & -1.76 \times 10^{-3} & -9.0 \times 10^{-2} & 2.0 \end{bmatrix} \begin{bmatrix} N_x^T \\ N_y^T \\ M_x^T \\ M_y^T \end{bmatrix} \quad (5.30)$$

The thermal stress resultants and moment resultants are calculated using eqs. (5.22) to be

$$\begin{aligned}
 N_x^T &= 8.22 \times 10^6 h \Delta T \\
 N_y^T &= 6.42 \times 10^6 h \Delta T \\
 M_x^T &= 8.64 \times 10^7 \frac{h^2}{2} \Delta T \\
 M_y^T &= -8.64 \times 10^7 \frac{h^2}{2} \Delta T
 \end{aligned} \quad (5.31)$$

Using the value of $h = 0.125$ mm and $\Delta T = 20 - 177 = -157$ °C in eq. (5.31) gives the following values:

$$\begin{aligned} N_x^T &= -161,318 \text{ N/m} \\ N_y^T &= -126,008 \text{ N/m} \\ M_x^T &= -106 \text{ N} \\ M_y^T &= 106 \text{ N} \end{aligned} \quad (5.32)$$

Substituting into eq. (5.30) gives

$$\begin{aligned} \epsilon_x^0 &= -1.58 \times 10^{-3} \\ k_x &= -1.269 \end{aligned} \quad (5.33)$$

The strain at any point across the thickness of the laminate is given as

$$\begin{aligned} \epsilon_x &= \epsilon_x^0 + z k_x \\ \text{or} \\ \epsilon_x &= -1.58 \times 10^{-3} - 1.269z \end{aligned} \quad (5.34)$$

The neutral surface is where the strain is equal to 0:

$$z = \frac{1.58 \times 10^{-3}}{-1.269} = -1.245 \text{ mm}$$

This is located about 10 layers above the mid-plane and is within the 0° layers. The location of the neutral surface is shown in Figure 5.13b.

At the concave surface of the laminate (face of the 90° layers, Figure (5.13b)), $z = 2.5$ mm, the strain is

$$\epsilon_x = -4.75 \times 10^{-3} = -4,750 \mu\epsilon.$$

At the convex surface of the laminate (face of the 0° layers), $z = -2.5$ mm, the strain is

$$\epsilon_x = 1.593 \times 10^{-3} = 1,593 \mu\epsilon$$

At the interface between the 0° layers and 90° layers, $z = -0.5$ mm, the strain is

$$\epsilon_x = -0.946 \times 10^{-3} = -946 \mu\epsilon$$

It can be seen that all of the 90° layers are under compressive strains, while some of the 0° layers (about 6 layers) are under compressive strains, and the rest (about 10 layers) are under tensile strain. The fact that the 90° layers are under compressive strain is good for mechanical loading, as shown below.

Strains due to mechanical load

Equation (5.30) can be used to write the strain–stress resultant for the case of mechanical load as follows:

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \kappa_x \\ \kappa_y \end{bmatrix} = 10^{-3} \begin{bmatrix} 7.5 \times 10^{-6} & -1.97 \times 10^{-7} & 3.7 \times 10^{-3} & -3.63 \times 10^{-5} \\ -1.97 \times 10^{-7} & 3.6 \times 10^{-6} & -3.42 \times 10^{-5} & -1.76 \times 10^{-3} \\ 3.7 \times 10^{-3} & -3.42 \times 10^{-5} & 3.01 & -9.0 \times 10^{-2} \\ -3.63 \times 10^{-5} & -1.76 \times 10^{-3} & -9.0 \times 10^{-2} & 2.0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ M_x^M \\ 0 \end{bmatrix}$$

The strain at any point can be written as

$$\varepsilon_x = (3.7 \times 10^{-6} + 3.01 \times 10^{-3} z) M_x^M \quad (5.35)$$

Combining eqs. (5.34) and (5.35), the combined strains due to residual thermal stresses and applied mechanical load are

$$\varepsilon_x = -1.58 \times 10^{-3} + 3.7 \times 10^{-6} M_x^M + (-1.269 + 3.01 \times 10^{-3} z) M_x^M \quad (5.36)$$

The neutral axis corresponds to when the strain is zero, given by

$$z_e = \frac{1.58 \times 10^{-3} - 3.7 \times 10^{-6} M_x^M}{-1.269 + 3.01 \times 10^{-3} M_x^M} \quad (5.37)$$

It is of interest to determine the strain ε_x at the concave surface of the 90° layers (inner surface of the leaf spring). The strain due to manufacturing using 4DPC was given above as $-4,750 \mu\varepsilon$. The strain due to mechanical loading can be obtained from eq. (5.36). The value of z to be used can be obtained from eq. (5.37) and the thickness of the 90° layers as follows:

The maximum bending moment occurs at the mid-length of the beam. For a beam that is 24 in (609.6 mm) long, the maximum load, as given by Figure 5.14, is 200 N. The maximum bending moment is

$$M_x = (0.5)(200 \text{ N})(0.5)(60.96 \text{ cm}) = 30.48 \text{ N m}$$

The location of the elastic axis is given by eq. (5.37):

$$z_e = -1.246 \text{ mm}$$

The z value at the concave surface of the 90° layers is

$$z = z_e + mh = -1.246 + 24(0.125) = 1.654 \text{ mm}$$

Substituting the values of M_x and z into eq. (5.36) gives the strain due to mechanical loading:

$$\varepsilon_{X-\text{Mechanical}} = 274 \times 10^{-6}$$

The combination of strain due to manufacturing and mechanical loading is

$$\varepsilon_{X-\text{Combined}} = -4,750 + 274 = -4,476 \mu\varepsilon$$

It can be seen that the combined strain due to manufacturing and mechanical loading is still highly compressive. This explains the good fatigue performance of the composite leaf spring made by 4DPC.

5.4 Letters of the alphabet

One application that shows the capability and flexibility of the technique of 4DPC is the formation of letters of the alphabet using composite materials and only a flat mold [6].

This is shown through the procedure to manufacture the first four letters: a, b, c, and d.

5.4.1 Principles of the mechanism

The formation of the letters with complex geometries utilizes the following four mechanisms:

1. Symmetric laminates should be flat:

Laminates with symmetric lay-up sequences such as $[0/90]_s$ or $[0_m/90_n]_s$ should remain flat after curing and cooling to room temperature. These laminates have been used by the composites community for many years in making various engineering structures such as airplane and automotive components. These types of laminates serve as the base for the letter and are used in all letters in this section.

2. Unidirectional laminates should also be flat:

In laminates for all the letters in this section, there are regions where the laminate has a combination of $[0]_1$ and $[0]_3$. This combination provides simply the total laminate of $[0]_4$, which means a laminate with four layers, all along the 0° direction. Upon curing and cooling to room temperature, this laminate should also remain flat.

3. Cross ply laminates show curvature:

Cross-ply laminates are those consisting of layers at 0° orientation and 90° orientation. The laminate sequence has the form $[0_m/90_n]$, where m and n are integers. For example, in Figure 5.14 for the letter “a,” there are regions where the laminate has the lay-up sequence $[0/90_3]$, where $m = 1$ and $n = 3$. The principle for the changing of shape from flat to curved has been shown in reference [1].

4. Use of a nonstick separator to prevent bonding:

There are regions in some of the letters that should be separated after curing and cooling. Consider the case of the letter “a” in Figure 5.14. The red line shows a nonstick caul plate. Its function is to prevent bonding between the base and the laminate above. At the beginning, during the deposition of the layers, all layers are flat and are on top of each other. Since the prepregs are adhesive, they need to be separated such that when they cure and solidify, the layers do not bond to each other. This allows the rise of the majority of the letter “a” above the base plate.

Using the above four mechanisms, the lay-up sequences for a few different letters are presented below.

5.4.2 Letter “a”

The letter “a” can be considered to consist of two parts. A curved part on the left can be approximated as the left half of a circle. The right part is an inclined straight part. The half circle can be made using a lay-up sequence of the type $[0_m/90_n]$. In order to accommodate manufacturing, the straight part is approximated as a portion of a circular part. As such, the lay-up arrangement for this letter can be as follows:

- A flat base plate is made of a symmetric laminate with a lay-up sequence $[0/90]_s$. This is shown as the bottom blue part.
- A thin sheet of nonstick material (brass shim or caul plate) is placed on top of the base plate at the left end, starting at a point about 2 in from the left end. This is shown as the thin red line.
- The third level of lay-up consists of three parts. On top of the brass shim is a laminate made of $[90]_3$ lay-up, with a length of 20 in (508 mm). The right end of this laminate is at the same location as the right end of the brass shim below. At the left end of the $[90]_3$ laminate is a $[0]_3$ laminate with a length of 2 in (50.8 mm). Also, at the right end of the $[90]_3$ laminate is another $[0]_3$ laminate with a length of 2 in (50.8 mm).
- The fourth level of the lay-up is a layer with the lay-up $[0]_1$. This has a length of 24 in (609.6 mm) and spans the whole length of the third-level layer.
- The fifth level of the lay-up is a second brass shim with a length of 10.5 in (266.7 mm). The left end of the brass shim is located 8 in (203.2 mm) from the left end of the $[90]_3$ layer in level 3.
- The sixth level of the lay-up is another layer with a lay-up sequence $[0]_1$. It has a length of 12.5 in (317.5 mm). Its left end is at the same position as the left end of the second brass shim.
- The seventh and last level of the lay-up is a layer consisting of 3 plies. Starting from the left end (its left end is at the same location as the left end of the second brass shim) is a layer with a lay-up sequence $[0]_3$. It has a length of 4 in (101.6 mm). Next is a laminate with a lay-up $[90]_3$ with a length of 6.5 in (165.1 mm). Finally, there is a laminate with a lay-up $[0]_3$, with a length of 2 in (50.8 mm).

Note that in the lay-up arrangement in Figure 5.14, the two main actors are the combinations $[90_3/0]$ at the lower left, and $[0/90_3]$ at the upper right. The $[90_3/0]$ will bend concave downward, while the $[0/90_3]$ will bend concave upward. The presence of the brass shims allows these to separate from the rest of the structure.

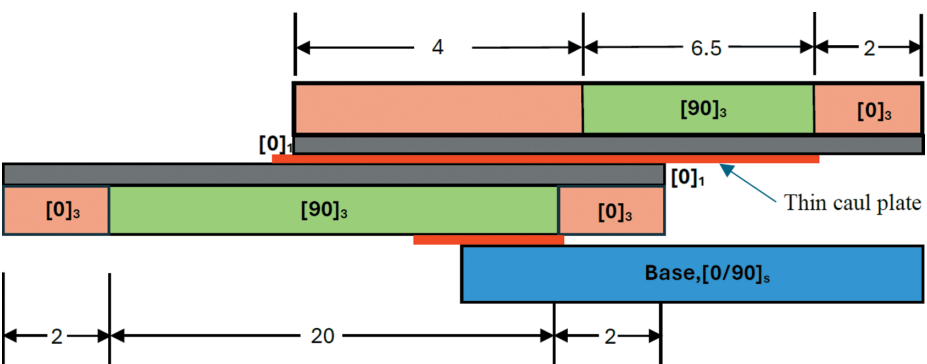


Figure 5.14: Lay-up for letter “a” [6].

Figure 5.15 shows the letter “a” after curing and cooling to room temperature.



Figure 5.15: Letter “a” after curing and cooling to room temperature [6].

5.4.3 Letter “b”

The letter “b” can be considered to consist of two parts. The first is a curved part on the right. This can be considered to be a half circle. The second is an inclined straight

part on the left. As such, this letter has similar components to the letter “a” except there is a flip over, and the straight part in “b” is longer than in “a.” Figure 5.16 shows the lay-up for the letter “b.” The lay-up arrangement for this letter is as follows:

- First, there is a base plate 13.5 in (342.9 mm) long (blue region). It is a symmetric laminate with a lay-up sequence $[0/90]_s$.
- A piece of thin brass shim is placed on top of the right part of the base plate (red line).
- The third level has a thickness of 3 plies. It starts with a 2 in (50.8 mm) long $[0]_3$ laminate on the left. This is followed by a 20 in (508 mm) long $[90]_3$ laminate. On the right side is a 2 in (50.8 mm) long $[0]_3$ laminate.
- The fourth level is a 24 in (609.6 mm) single unidirectional layer (brown line).
- The fifth level is a second brass shim (second red line).
- The sixth level is a single unidirectional layer of $[0]_1$ that is 18.5 in (469.9 mm) long (brown line).
- The seventh (and last) level has a thickness of 3 plies. It starts with the $[0]_3$ laminate that is 10 in (508 mm) long on the right side. This is followed by a $[90]_3$ laminate that is 6.5 in (165.1 mm) long. This is followed by another $[0]_3$ laminate that is 2 in (50.8 mm) long.

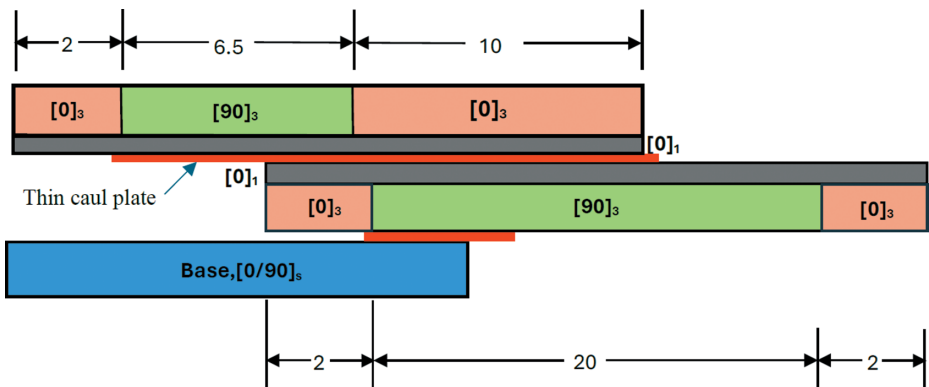


Figure 5.16: Lay-up arrangement for the letter “b.”

Figure 5.17 shows the final letter “b” after curing and cooling to room temperature.

5.4.4 Letter “C”

Considering the shape of the letter “C,” it is concave toward the right-hand side. The letter is not a circle since it has portions that are curved and other portions that are not. As such, the lay-up sequences need to vary along the curvilinear length of the letter. For the curved part, an unsymmetric laminate such as $[0/90]$ would do. For the other parts,



Figure 5.17: Letter “b” after curing and cooling to room temperature.

straight laminates would be used. In order to support this laminate, a base plate needs to be used. The base plate is bonded to the base of the letter “C.” Since everything is deposited flat initially, there should be a way to separate the prepreps of the base plate from the prepreps of the letter “C.” This is done by using a nonstick brass shim.

With the above considerations, the lay-up sequence for the letter C is shown in Figure 5.18. The sequence for the lay-up is as follows:

- A base plate consisting of four layers, with the lay-up sequence of $[0/90]_s$, a length of 12.8 in (325.1 mm), and a width of 3 in (76.2 mm) is made. This base plate has a symmetric lay-up and will remain flat after curing and cooling to room temperature. This serves as the base for the letter.

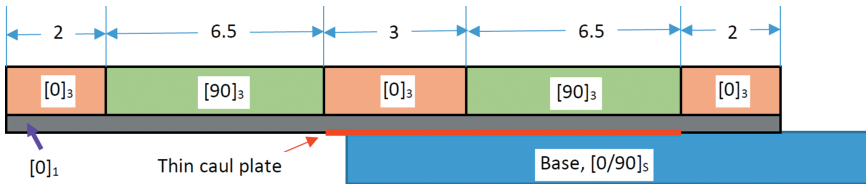


Figure 5.18: Lay-up sequence for the letter C [6].

- A thin brass shim (denoted as caul plate in Figure 5.18) with a width of 3 in (76.2 mm) and a length of 9.5 in (241.3 mm) is placed on top of the base plate. The right end of the brass shim starts at a location that is 3.5 in (88.9 mm) from the right end of the base plate. The brass shim is nonstick and serves to separate the rest of the composite lay-up from the base plate.
- The next layer is a unidirectional layer with the lay-up sequence $[0]_1$. This layer has a width of 3 in (76.2 mm) and a length of 20 in (508 mm). This layer will bond

with the base plate over a distance of 2 in (50.8 mm) on the right side, but will be separated from the base plate over the length of the brass shim.

- The next layer has a thickness of 3 plies. Its lay-up sequence varies along the length. Starting from the left end is a $[0]_3$ layer with a length of 2 in (50.8 mm). Together with the $[0]_1$ layer, this will make a flat laminate of $[0]_4$ stacking after curing and cooling to room temperature. Next on the right is a $[90]_3$ layer with a length of 6.5 in (165.1 mm). Together with the $[0]_1$ layer, this will make an unsymmetric laminate that will curl with the $[90]_3$ layers on the concave side. Next on the right is another $[0]_3$ stack with a length of 3 in (76.2 mm). This together with the $[0]_1$ layer will make a flat laminate of $[0]_4$ after curing and cooling. On the right of this is another $[90]_3$ stack with a length of 6.5 in (165.1 mm). This together with the $[0]_1$ layer below will make another unsymmetric laminate that will curl upon curing and cooling. Finally, on the right end, there is another stack of $[0]_3$ with a length of 2 in (50.8 mm).
- The whole lay-up sequence will provide a letter C with three flat regions and two curved regions, as shown in Figure 5.19.



Figure 5.19: Letter C after curing and cooling to room temperature [6].

5.4.5 Letter “d”

The letter “d” can be considered to consist of two parts. A circular part on the left and a straight vertical part on the right are present. The circular part on the left is concave inward. This can be done by having the 90° layers below the 0° layers. The straight part on the right can be done by having a long portion that consists of 0° layers only. This part needs to be pulled to the right by an unsymmetric laminate with 90° layers on top of the 0° layers.

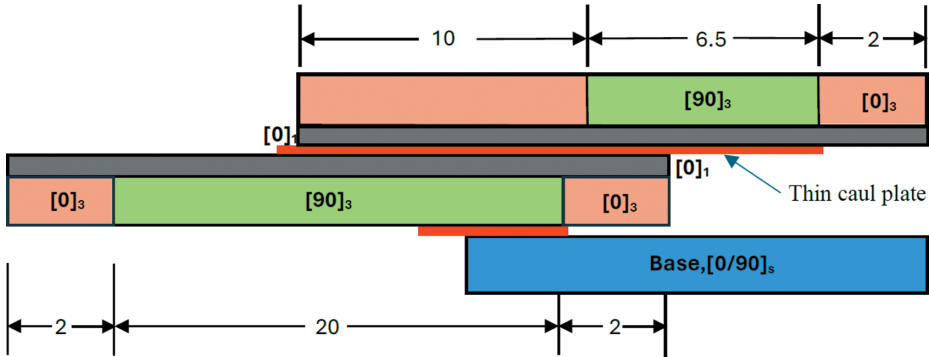


Figure 5.20: Lay-up for letter “d.”



Figure 5.21: Letter “d” after curing and cooling to room temperature.

Figure 5.20 shows the lay-up for the letter “d.” Figure 5.21 shows the letter “d” after curing and cooling to room temperature.

It can be noted that other letters in the alphabet can also be made using similar considerations and techniques. Even though the illustrations here are for the case of the letters in the alphabet, the experience learned here can be used to make other types of structures.

5.5 Composite structures made of fiber orientations other than 0° or 90°

Laminates made of fiber orientation other than 0° or 90° tend to have twisting curvature, in addition to bending curvature. Among engineering structures, there are those that contain twisted plates or shells. One example is the blade of a vertical wind turbine, as shown in Figure 5.22. The structure (as obtained from a commercial source) is made of thin sheets of metal or composites. The sheets are initially flat, and they are bent into the twisted shape. The bent piece is then bolted to the fixed frame to provide the shape. Bending requires brute force, and sometimes distortion and cracks occur.



Figure 5.22: Twisted blade in a vertical wind turbine [8].

The method of 4DPCs can provide twisted composite structures. This can be done by having layers with fiber angles that differ from each other by a quantity different from 90° . For example, a laminate with a lay-up sequence $[0/\theta]$, where θ is any angle between 0° and 90° , or a combination of these types of laminates may provide a twisted shape. In the following, the behavior of a few laminates made of fiber orientations other than 0° or 90° is examined. This can serve as a guide to study the behavior of laminates of other stacking sequences or their combinations.

5.5.1 Laminate made of [45°/-45°] stacking sequence

This laminate is just the [0/90] laminate rotated by 45°. As such, one can tell right away that the deformed configuration is a portion of a right circular cylinder with its axis oriented at 45° with respect to the x -axis. Figure 5.23 shows the fiber orientation for layer 1 and layer 2. Figure 5.24 shows the predicted deformed shape.

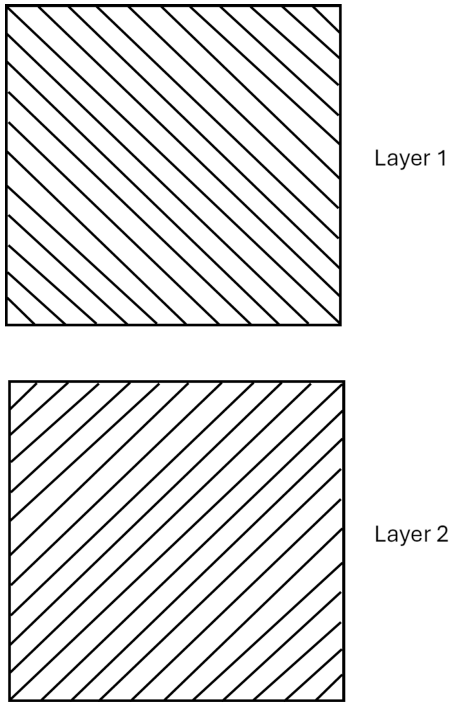


Figure 5.23: Fiber orientation in the flat blank at 45° and -45°.

Even though the deformed shape can be easily predicted, it is useful to go through the steps of laminate theory to see if the same result can be obtained. If proven, this can be extended to laminates made of other types of lay-up angles.

The material properties in Table 1.1 are used in the following calculations. The on-axis stiffness coefficients are:

$$Q_{11} = 155.7 \text{ GPa} \quad (5.38a)$$

$$Q_{22} = 12.16 \text{ GPa} \quad (5.38b)$$

$$Q_{12} = 3.02 \text{ GPa} \quad (5.38c)$$

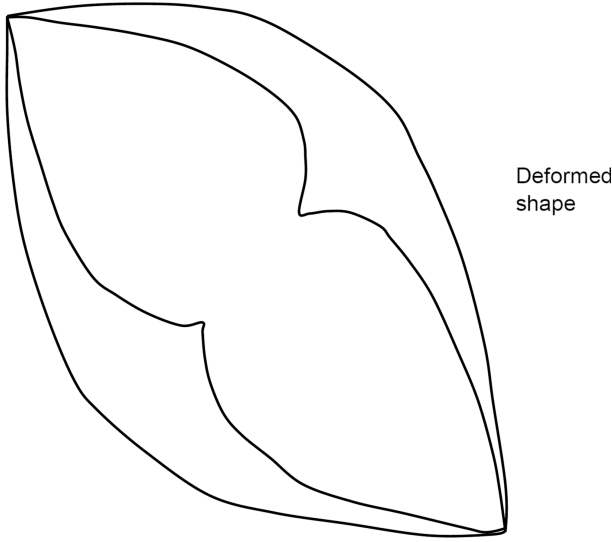


Figure 5.24: Predicted deformed shape.

$$Q_{66} = 4.40 \text{ GPa} \quad (5.38d)$$

Substituting the values in eq. (5.38) into eq. (1.24) gives the values for the off-axis stiffness coefficients:

For $\theta = 45^\circ$:

$$\overline{Q}_{11} = 0.25Q_{11} + 0.5(Q_{12} + 2Q_{66}) + 0.25Q_{22} \quad (5.39a)$$

$$\overline{Q}_{12} = 0.25(Q_{11} + Q_{22} - 4Q_{66}) + 0.5 Q_{12} \quad (5.39b)$$

$$\overline{Q}_{16} = 0.25(Q_{11} - Q_{22}) \quad (5.39c)$$

$$\overline{Q}_{22} = 0.25Q_{11} + 0.5(Q_{12} + 2Q_{66}) + 0.25Q_{22} = \overline{Q}_{11} \quad (5.39d)$$

$$\overline{Q}_{26} = 0.25(Q_{11} - Q_{22}) \quad (5.39e)$$

$$\overline{Q}_{66} = 0.25 (Q_{11} + Q_{22} - 2Q_{12}) \quad (5.39f)$$

It can be shown that the off-axis stiffness coefficients for $\theta = -45^\circ$ are the same as those for $\theta = 45^\circ$, except that $\overline{Q}_{16} = \overline{Q}_{26} = 0.25(Q_{22} - Q_{11})$, which they are opposite in sign to those for $\theta = -45^\circ$.

For the off-axis coefficients of thermal expansion, eq. (1.25) can be used to give:

For $\theta = 45^\circ$:

$$\alpha_x = 0.5(\alpha_1 + \alpha_2) \quad (5.40a)$$

$$\alpha_y = 0.5(\alpha_1 + \alpha_2) = \alpha_x \quad (5.40b)$$

$$\alpha_{xy} = \alpha_1 - \alpha_2 \quad (5.40c)$$

For $\theta = -45^\circ$

$$\alpha_x = 0.5(\alpha_1 + \alpha_2) \quad (5.40d)$$

$$\alpha_y = 0.5(\alpha_1 + \alpha_2) = \alpha_x \quad (5.40e)$$

$$\alpha_{xy} = \alpha_2 - \alpha_1 \quad (5.40f)$$

Note that when θ changes the sign from plus to minus, the sign of the α_{xy} changes. This is the only change in all the off-axis coefficients.

Substituting the values in (5.38) into (5.39) gives (for both $\theta = 45^\circ$ and $\theta = -45^\circ$)

$$\overline{Q_{11}} = 47.88 \text{ GPa} = \overline{Q_{22}} \quad (5.41a)$$

$$\overline{Q_{12}} = 39.08 \text{ GPa} \quad (5.41b)$$

$$\overline{Q_{16}} = 0 = \overline{Q_{26}} \quad (5.41c)$$

$$\overline{Q_{66}} = 40.46 \text{ GPa} \quad (5.41d)$$

Using values of the coefficients of thermal expansion from Table 1.1 in eqs. (5.40) gives

$$\alpha_x = 12.14 \times 10^{-6} = \alpha_y \text{ (for both } \theta = 45^\circ \text{ and } \theta = -45^\circ) \quad (5.42a)$$

$$\alpha_{xy} = 24.32 \times 10^{-6} \text{ for } \theta = 45^\circ \quad (5.42b)$$

$$\alpha_{xy} = -24.32 \times 10^{-6} \text{ for } \theta = -45^\circ \quad (5.42c)$$

Using eq. (1.28) with layer thickness $h = 0.125 \text{ mm}$ gives

$$A_{11} = 11.97 \times 10^6 \text{ N/m} = A_{22} \quad (5.43a)$$

$$A_{12} = 9.77 \times 10^6 \text{ N/m} \quad (5.43b)$$

$$A_{66} = 10.12 \times 10^6 \text{ N/m} \quad (5.43c)$$

$$B_{ij} = 0 \quad (5.44)$$

$$D_{11} = 0.0623 \text{ N m} = D_{22} \quad (5.45a)$$

$$D_{12} = 0.0509 \text{ N m} \quad (5.45b)$$

$$D_{66} = 0.0527 \text{ N m} \quad (5.45c)$$

Using eqs. (1.29d)–(1.29f) gives

$$M_x^T = M_y^T = 0 \quad (5.46a)$$

$$M_{xy}^T = \bar{Q}_{66}(\alpha_2 - \alpha_1)\Delta T \quad h^2 = 2.41 \text{ N} \quad (5.46b)$$

The in-plane equations are decoupled from the out-of-plane equation. The out-of-plane equation is

$$M_{xy}^T = D_{66}k_{xy} \quad (5.47)$$

Giving:

$$k_{xy} = \frac{2.41}{0.0527} = \frac{45.8}{\text{m}} = \frac{0.458}{\text{cm}} \quad (5.48)$$

Using the transformation to obtain the principal curvatures:

$$K_1 = \frac{K_x + K_y}{2} + \sqrt{\left(\frac{K_x - K_y}{2}\right)^2 + \left(\frac{K_{xy}}{2}\right)^2} \quad (5.49a)$$

$$K_2 = \frac{K_x + K_y}{2} - \sqrt{\left(\frac{K_x - K_y}{2}\right)^2 + \left(\frac{K_{xy}}{2}\right)^2} \quad (5.49b)$$

$$\tan 2\gamma = \frac{K_{xy}}{2(K_x - K_y)} \quad (5.49c)$$

gives

$$K_1 = \frac{1}{2}K_{xy} = \frac{0.229}{\text{cm}} = -K_2 \quad (5.50a)$$

$$\gamma = 45^\circ \quad (5.50b)$$

As such, the principal radius of curvature is $R = 4.37 \text{ cm}$, and the principal curvature direction is at 45° with respect to the x -axis, as shown in Figure 5.24.

5.5.1.1 Finite element method

The finite element analysis was carried out using the software Abaqus. The elements used are S4R quadrilateral elements, suitable for capturing bending and large deformations. The plate dimensions are 6 in $\times$ 6 in (15.25 cm $\times$ 15.25 cm). Temperature dependence of the material properties is taken into account. The plate is modeled with 484 uniform square elements, as shown in Figure 5.25.

In order to avoid rigid body motion, a point in the structure is fixed. The usual method is to fix the center point. However, the result gives a saddle shape, as shown in Figure 5.26. The saddle shape indicates that there are equal and opposite curvatures along the x direction and the y direction. This is the result of equal competition for bending along the two directions, as explained in reference [10]. In reality, at room tempera-

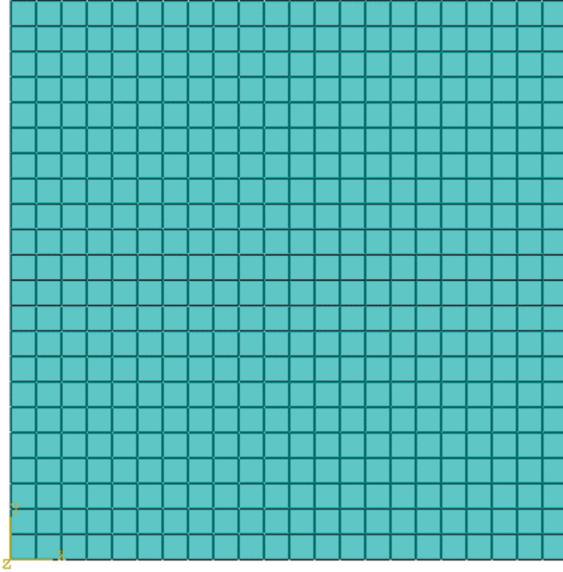


Figure 5.25: Uniform square elements.

ture, the shape is a portion of a right circular cylinder, with the axis of principal curvature rotated by 45° , as shown in Figure 5.24. In order to obtain this shape, it is necessary to offset the equality between the two directions. This is done by shifting the fixed point to one node along one of the diagonals of the samples, as shown in Figure 5.27. In Figure 5.27, the fibers in one layer are along the 45° direction, and the fibers in the other layer are along the -45° direction. The fixed point is shifted from O to O_1 . The finite element result using this shifted fixed point is shown in Figure 5.28. The shape in this figure is similar to the predicted shape shown in Figure 5.24.

5.5.2 Laminate made of $[0/45]$ stacking sequence

It is expected that this laminate will exhibit both bending and twisting curvatures.

Figure 5.29 shows the final configuration (after curing and cooling to room temperature) of a laminate made of the lay-up sequence $[0/45]$. A formulation using laminate theory (presented below) shows three in-plane strains and three curvatures (K_x , K_y , and K_{xy}). However, from the appearance, the laminate exhibits only two curvatures: one bending curvature and one twisting curvature. The difference is due to the dominance of one bending curvature over the other as the laminate is cooled from cure temperature to room temperature, as presented earlier in Chapter 1. It is important to determine the accuracy of the analytical methods (laminate theory and finite element methods) in calculating the deformed configuration vis-à-vis measured deformation.

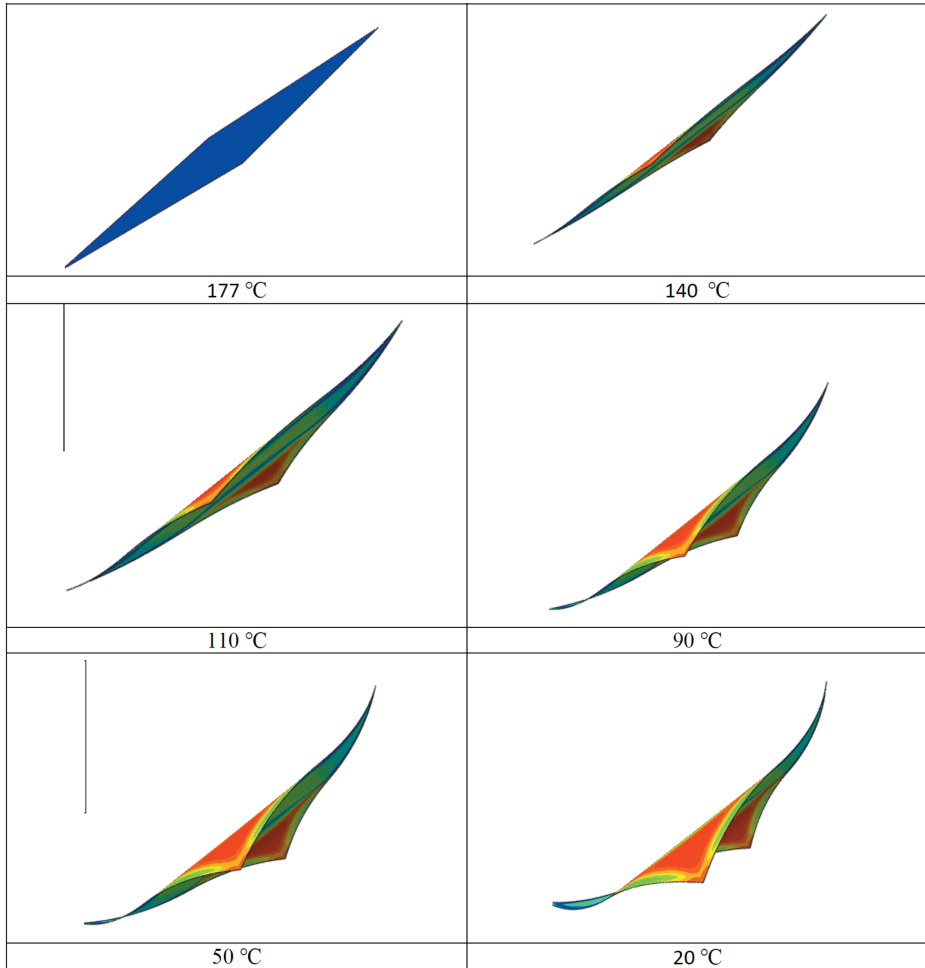


Figure 5.26: Saddle shape as given by the finite element method (center point fixed).

It is of interest to determine the direction of the principal curvature and its magnitude. This will be done using laminate theory and finite element analysis, as shown below.

5.5.2.1 Laminate theory

The $\bar{Q}_{ij}$ for the 45° layer are given in eqs. (5.41), and its off-axis coefficients of thermal expansion are given in eq. (5.42). The $\bar{Q}_{ij}$ for the 0° layer are the same as the on-axis values. Using these, one has:

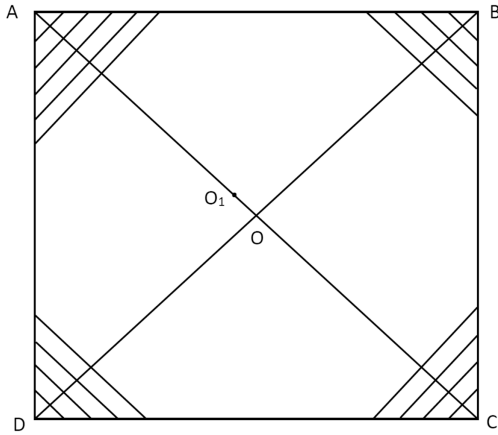


Figure 5.27: The fixed point is shifted to one node along one of the diagonals (from O to O_1).

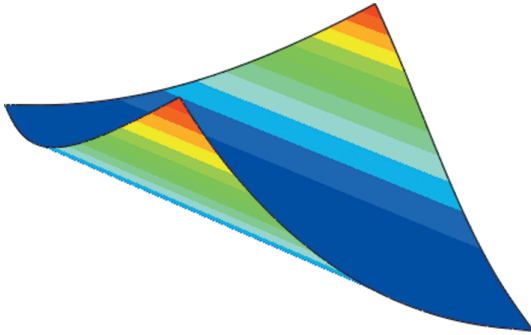


Figure 5.28: Final shape (cylindrical bending) of the $[45/-45]$ laminate using a shifted fixed point from O to O_1 .

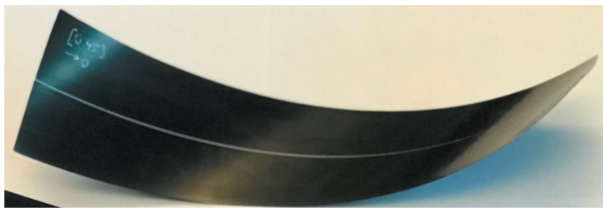


Figure 5.29: Configuration of laminate made of lay-up sequence $[0/45]$: CYCOM 977-2-5-12K HTS-145 – 11.5 in $\times$ 3.5 in (292.1 mm $\times$ 88.9 mm) [7].

$$A_{11} = 25.45 \times 10^6 \text{ N/m} \quad (5.51a)$$

$$A_{12} = 5.26 \times 10^6 \text{ N/m} \quad (5.51b)$$

$$A_{16} = 4.48 \times 10^6 \text{ N/m} \quad (5.51c)$$

$$A_{22} = 7.51 \times 10^6 \text{ N/m} \quad (5.51d)$$

$$A_{26} = 4.48 \times 10^6 \text{ N/m} \quad (5.51e)$$

$$A_{66} = 5.61 \times 10^6 \text{ N/m} \quad (5.51f)$$

$$B_{11} = -842 \text{ N} \quad (5.52a)$$

$$B_{12} = 282 \text{ N} \quad (5.52b)$$

$$B_{16} = 280 \text{ N} \quad (5.52c)$$

$$B_{22} = 279 \text{ N} \quad (5.52d)$$

$$B_{26} = 280 \text{ N} \quad (5.52e)$$

$$B_{66} = 282 \text{ N} \quad (5.52f)$$

$$D_{11} = 0.133 \text{ N m} \quad (5.53a)$$

$$D_{12} = 0.0274 \text{ N m} \quad (5.53b)$$

$$D_{16} = 0.0234 \text{ N m} \quad (5.53c)$$

$$D_{22} = 0.039 \text{ N m} \quad (5.53d)$$

$$D_{26} = 0.0234 \text{ N m} \quad (5.53e)$$

$$D_{66} = 0.0292 \text{ N m} \quad (5.53f)$$

$$N_x^T = -4,293 \text{ N} \quad (5.44a)$$

$$N_y^T = -9,434 \text{ N} \quad (5.44b)$$

$$N_{xy}^T = 2,218 \text{ N} \quad (5.44c)$$

$$M_x^T = -0.275 \text{ N m} \quad (5.45a)$$

$$M_y^T = -0.677 \text{ N m} \quad (5.45b)$$

$$M_{xy}^T = 0.139 \text{ N m} \quad (5.45c)$$

The relationship between strains and load resultants is

$$\begin{bmatrix} -4,293 \\ -9,434 \\ 2,218 \\ -0.275 \\ -0.677 \\ 0.139 \end{bmatrix} = \begin{bmatrix} 25.09 \times 10^6 & 5.26 \times 10^6 & 4.48 \times 10^6 & -865 & 282 & 280 \\ 5.26 \times 10^6 & 7.51 \times 10^6 & 4.48 \times 10^6 & 282 & 279 & 280 \\ 4.48 \times 10^6 & 4.48 \times 10^6 & 5.61 \times 10^6 & 280 & 280 & 282 \\ -865 & 282 & 280 & 0.131 & 0.027 & 0.023 \\ 282.0 & 279 & 280 & 0.027 & 0.039 & 0.023 \\ 280 & 280 & 282 & 0.023 & 0.23 & 0.0292 \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} \quad (5.46)$$

The 6×6 matrix in eq. (5.46) is ill-conditioned, and it is difficult to invert. Instead, the following procedure via inverse by partitioning is used.

Equation (5.46) can be written in partitioned form as follows:

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ K \end{bmatrix} = \begin{bmatrix} N \\ M \end{bmatrix} \quad (5.47)$$

where each letter represents a matrix.

Expanding gives

$$A\varepsilon + BK = N \quad (5.48a)$$

$$B\varepsilon + DK = M \quad (5.48b)$$

Solving gives

$$K = [A^{-1}B - B^1D]^{-1} [A^{-1}N - B^{-1}M] \quad (5.49)$$

Substituting K into either eq. (5.48a) or (5.48b) will give ε .

Using the MATLAB program (Appendix), the curvatures for laminates of the lay-up sequence $[0/\theta]$ were obtained. These are given in Table 5.3. (Note the y represents angle between principal direction and x axis).

Table 5.3: Curvatures for laminates of the type $[0/\theta]$.

θ (degree)	K_x (m^{-1})	K_y (m^{-1})	K_{xy} (m^{-1})	K_1 (m^{-1})	K_2 (m^{-1})	γ (degree)
15	0.90	-60.4	11.4	1.42	-60.9	2.7
25	2.62	-70.5	22.1	4.35	-72	4.3
35	2.20	-62.1	33.8	6.37	-66.3	7.4
45	-2.20	-37.4	36.8	5.62	-45.2	13.8
55	-7.63	-15.2	31.1	5.87	-28.7	32.0
65	-11.30	-7.10	27.7	4.80	-23.2	-36.6
70	-12.70	-6.71	27.5	4.36	-23.8	-32.3
75	-13.98	-6.98	26.1	3.03	-24.0	-30.9
85	-16.60	-6.98	26.1	2.11	-25.7	-26.8
-70	-12.63	-6.71	-27.5	4.36	-23.8	32.3

The principal curvature and its orientation can be calculated using eqs. (5.49).

The following observations are made for laminates of lay-up sequence type $[0/\theta]$:

- For the range of θ from 15° to 85° , according to the laminate theory, the laminate takes on a saddle shape, with stronger curvature along the two directions. However, due to the dominance of curvature along the two directions, the final shape at room temperature will involve bending along the principal axis 2.
- The variation of the orientation of the principal axis of curvature (angle γ) increases as the θ angle increases from 15° to 55° . Between a θ angle of 55° and 65° , there is a switch in sign in the orientation angle. Then, the magnitude of the orientation angle decreases as θ keeps on increasing. A graph of the variation of the orientation angle and θ is shown in Figure 5.30.
- When the θ angle switches from 70° to -70° , the values of the principal curvatures stay the same, but the orientation of the axes changes the sign.

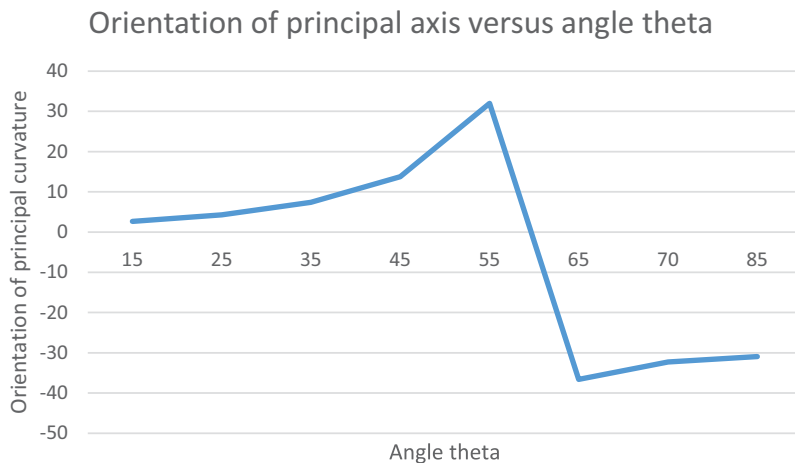


Figure 5.30: Variation of γ versus θ .

5.5.2.2 Finite element method

The finite element method was also used to determine the deformed configuration of the laminate as it cools from cure temperature to room temperature. The laminate was modeled with 576 fully integrated shell elements with a mesh size of 0.25 in in Abaqus. The material properties of the unitape were provided in Table 1.1. The center of the laminate was fixed. (Note that for the case of a rectangular plate, it was not necessary to shift the fixed point in order to create a dominance along one direction versus the other direction. This is because the rectangular shape already provides dominance for bending. This is different from the case of a square plate discussed earlier where the shift in the position of the fixed point was necessary.) For the range of temperature from 177 to 20°C , the analysis was carried out using the implicit non-

linear FEA solver. For example, the first increment of -10°C (from 177 to 167°C) was first imposed on the flat configuration. The analysis was carried out to determine the deformed configuration. This deformed configuration was then used as input for the next increment of thermal loading. The coordinates of the grid points at the final step (20°C) are tabulated in Table 5.4 (columns 5, 6, and 7). The experimental values were measured using a laser coordinate measurement system.

In Table 5.4, columns 2, 3, and 4 present the coordinates of the grid points in the flat, undeformed configuration. Columns 5, 6, and 7 present the coordinates of the grid points in the deformed configuration using the finite element method. Columns 9, 10, and 11 present the coordinates of the deformed piece using the experimental method.

Table 5.4: Coordinates of different points as shown in Figure 5.31 in the initial and final configurations – dimensions in inches.

Points	Undeformed shape			FE			CMM		
	X	Y	Z	X	Y	Z	X	Y	Z
16	-1.99	-1.81	0.00	-1.99	-1.81	0.03	-1.99	-1.80	0.05
17	-1.99	-0.91	0.00	-1.99	-0.91	0.09	-1.98	-0.91	0.15
18	-1.99	0.00	0.00	-1.98	-0.01	0.18	-1.96	-0.02	0.31
19	-1.99	0.91	0.00	-1.96	0.88	0.31	-1.92	0.85	0.53
20	-1.99	1.81	0.00	-1.94	1.77	0.46	-1.85	1.69	0.82
21	-0.99	-1.81	0.00	-1.00	-1.81	0.00	-1.00	-1.80	0.01
22	-0.99	-0.91	0.00	-0.99	-0.90	0.01	-1.00	-0.90	0.02
23	-0.99	0.00	0.00	-0.99	0.00	0.05	-0.99	0.00	0.09
24	-0.99	0.91	0.00	-0.99	0.90	0.12	-0.98	0.89	0.22
25	-0.99	1.81	0.00	-0.98	1.80	0.22	-0.95	1.76	0.40
26	0.00	-1.81	0.00	0.00	-1.80	0.06	-0.01	-1.81	0.13
27	0.00	-0.91	0.00	0.00	-0.90	0.02	0.00	-0.91	0.03
28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	0.00	0.91	0.00	0.00	0.90	0.02	0.00	0.90	0.03
30	0.00	1.81	0.00	0.00	1.80	0.06	0.01	1.80	0.13
31	0.99	-1.81	0.00	0.98	-1.80	0.22	0.94	-1.80	0.41
32	0.99	-0.91	0.00	0.99	-0.90	0.12	0.98	-0.90	0.22
33	0.99	0.00	0.00	0.99	0.00	0.05	0.99	0.00	0.09
34	0.99	0.91	0.00	0.99	0.90	0.01	1.00	0.90	0.02
35	0.99	1.81	0.00	1.00	1.81	0.00	1.00	1.82	0.01
36	1.99	-1.81	0.00	1.94	-1.77	0.46	1.84	1.74	0.84
37	1.99	-0.91	0.00	1.96	-0.88	0.31	1.92	-0.87	0.55
38	1.99	0.00	0.00	1.98	0.01	0.18	1.96	0.02	0.32
39	1.99	0.91	0.00	1.99	0.91	0.09	1.98	0.91	0.15
40	1.99	1.81	0.00	1.99	1.81	0.03	1.99	1.83	0.04

Figures 5.31 show the deformed z coordinate variation with respect to undeformed x coordinates along the edges of the laminate (at $y = -1.81$ in and 1.81 in). The twisted

nature of the laminate is shown by the opposite slopes of the curves along the two edges of the laminate.

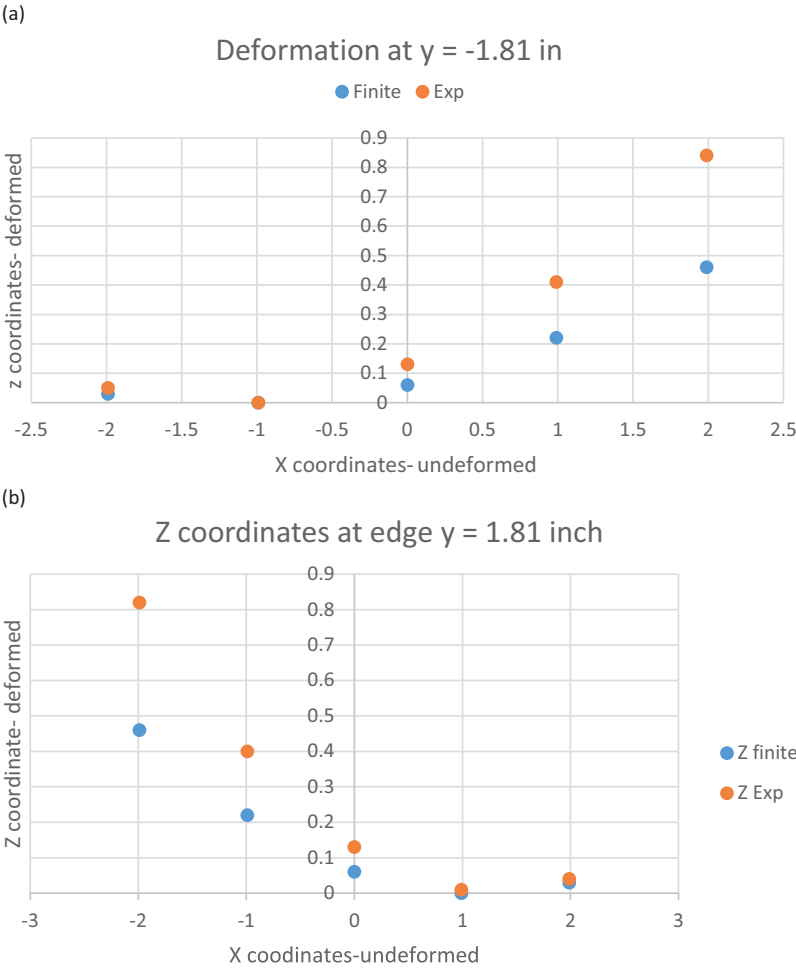


Figure 5.31: The z coordinates at edge (a) $y = -1.81$ in and (b) $y = 1.81$ in.

5.5.2.3 Discussion

- While laminate theory and finite element methods are useful for determining the shape of unsymmetric structures, care needs to be taken into account for the effects of physical aspects. These physical aspects are difficult to quantify since they are of a random nature, such as the variability of the fiber distribution in the microstructure, the distribution of the degree of cure, or microstructural de-

fects. Another physical effect is the difference in the degree of neighboring constraints between materials at the center of the plate and at the edge of the plate. Neglecting these physical aspects can lead to incorrect results.

- Figure 5.31 is plotted such that there is good agreement at one end of the plate but not good agreement at the other end. The plots can also be made to have good agreement at the center and relatively small nonagreements at the two ends. The reason for the nonagreement can be due to the edge effects.
- Laminates with a lay-up sequence such as $[0/70]$ can be considered equivalent to laminates with a lay-up sequence $[35/-35]$. The difference is in the rotation of the x -axis by 35° .
- The procedure developed can be used to study other laminates such as $[0_n/\theta]$ or $[0/\theta_n]$, where n is an integer.

5.5.2.4 Progression of the deformed shape during the cooling process

As presented in Chapter 1, there is a transition in the shape of the laminate from a saddle configuration to a straight cylindrical shape when the laminate is cooled from cure temperature to room temperature. The point of transition is called the bifurcation point. In Chapter 1, an example of a square laminate with layers consisting of 0° and 90° was presented. For these types of laminates, there is no twist and only bending curvatures exist. Here, the laminate has a lay-up sequence of $[0/45]$, and twist exists. It would be of interest to examine whether bifurcation also exists in these types of laminates. The following presents the configuration of the laminate at different temperature stages to see whether bifurcation exists.

For the laminate of $[0/45]$ lay-up sequence, the progression of the deformed shape for a rectangular piece of 12 in (304.8 mm) by 3 in (76.2 mm) is shown in Figures 5.32–5.34.

In Figure 5.32, the laminate was cooled down by 1% of the total thermal range. The fringe pattern shows three curvatures. There is a star pattern at the center of the laminate, indicating that there are two bending curvatures along two directions. The middle axis of this fringe pattern is inclined at an angle with respect to the edges of the laminate. This inclined angle is a result of the twisting curvature.

In Figure 5.33, the star pattern is still there, and the middle axis of the pattern is still making an angle with respect to the edges of the laminate. This pattern is not much different from that in Figure 5.32. This shows that the two bending curvatures and the twisting curvature are still in competition.

Figure 5.34 shows the deformation pattern when the laminate has already cooled to room temperature. The star pattern no longer exists. Instead, the blue pattern is more like a parallelism. The edges of this parallelism still make an angle with the edges of the laminate, showing the effect of the twist. As such, when this

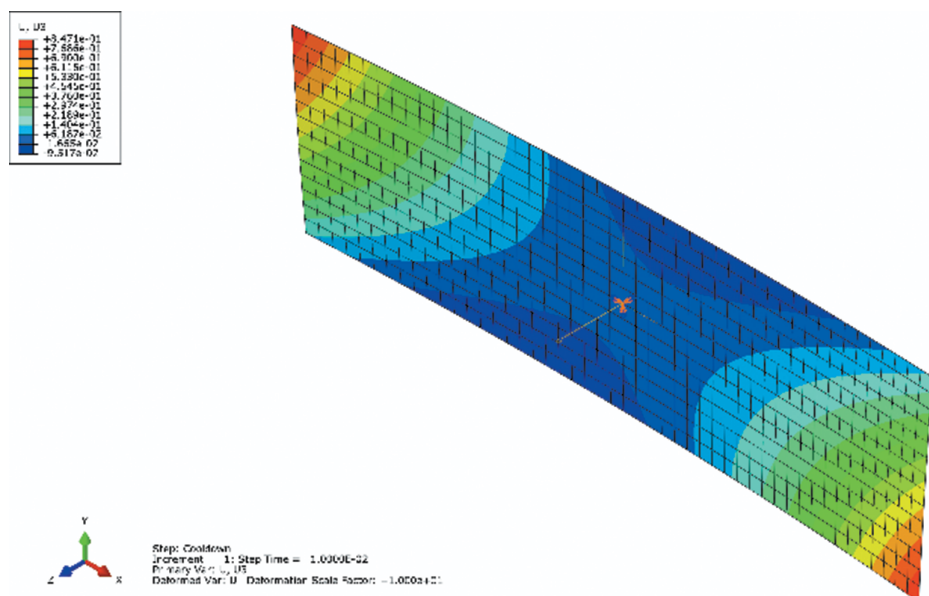


Figure 5.32: Deformation configuration of [0/45] laminate (12 in $\times$ 3 in) (304.8 mm $\times$ 76.2 mm) at 1% thermal loading.

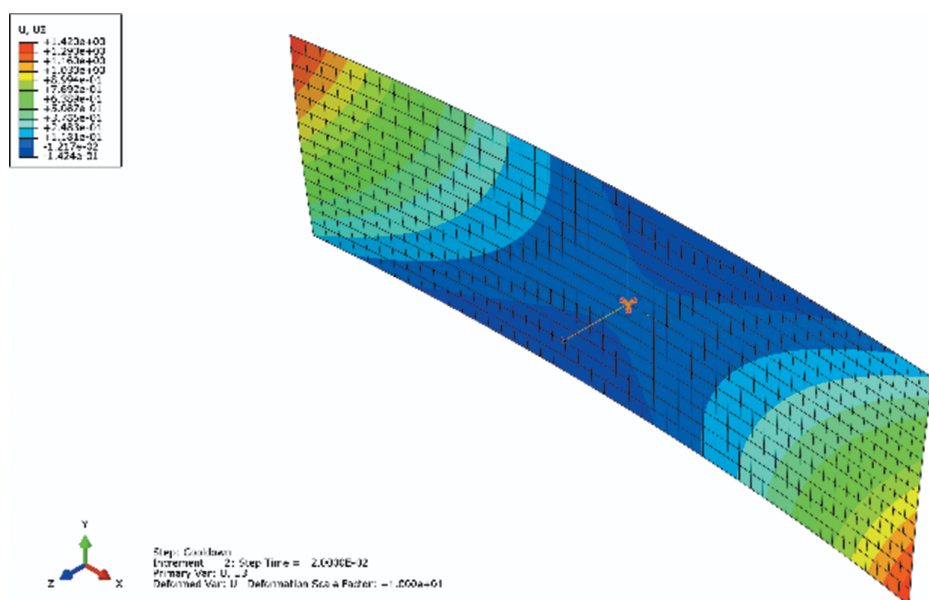


Figure 5.33: Deformation configuration of [0/45] laminate (12 in $\times$ 3 in) (304.8 mm $\times$ 76.2 mm) at 2% thermal loading.

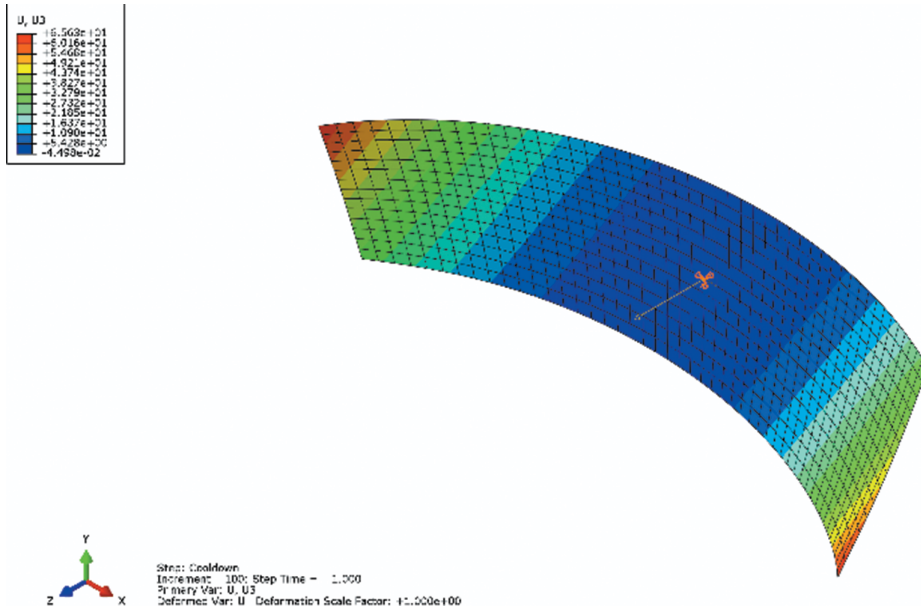


Figure 5.34: Deformation configuration of [0/45] laminate (12 in $\times$ 3 in) (304.8 mm $\times$ 76.2 mm) at 100% thermal loading.

laminate is cooled to room temperature, only two curvatures exist: one bending curvature and one twisting curvature.

There is a transition level of thermal loading where complete dominance occurs, and this can be much less than 100% (less than or equal to 10%). This transition level depends on the difference in bending stiffness along the competing directions. This stiffness is contained in the EI term, and depends on the properties and the shape of the initial flat structure. For example, for the [0/90] square piece, there is no dominance if there is no spatial variation in the properties of the material. For a rectangular laminate, the dominance is due to the difference in the dimensions of the two sides, in addition to whatever variabilities there may be within the structure.

5.6 Conclusion

The above sections show the method for the development of a few U-structures using the technique of 4DPCs. These are termed U-structures due to the free edges that occur. Due to the presence of the free edges, there is some variation in the curvature at regions close to the free edges and the curvature at the main central portion of the structure. Also, due to the presence of the free edges, the shapes of these structures are

sensitive to the effect of environmental conditions such as temperature and moisture. Whether this effect is desirable or undesirable depends on the particular situation.

On the positive side, the sensitivity of the shape to the hygrothermal environment can be used to make actuators or as sensors to detect changes in the environmental conditions. It can also be used for effective packaging in shipping packages to remote locations such as outer space or northern regions. Packages can be sent in flat form (which takes up less space). Upon arrival, thermal activation can be used to change the shape from flat to three-dimensional.

On the other hand, if one wants to have stability of the shape of the structure in operating conditions, this sensitivity is not desirable. One can constrain the movement of the structure by fixing the free edges. This results in the C-structures, as will be discussed in Chapter 6. One can also study the temperature range within which the change in shape is minimal and, as such, the structure can be operational. The following sections present studies that have been made on these aspects.

5.7 Environmental effects

Composite structures made by 4DPC depend on the interaction between plies of different fiber orientations as the temperature is reduced from the cure temperature to room temperature, causing a change from a flat shape to a curved shape. This is attractive from cost-saving and time-saving points of view since the method would not require a mold with a curved shape. However, for structures with free edges, such as U-structures, the change in environmental conditions during the operation of the final structures may affect the shape. The sensitivity to environmental conditions includes the variation of temperature and the absorption of moisture. As such, it is important to know the effect of these environmental conditions on the shape of U-structures. The effects of these environmental conditions are presented below [9].

5.7.1 Effect of heat

During the operating condition, the composite structure may be subjected to fluctuations in the temperature of the environment. The temperature fluctuation may range from -40 to 40 °C. Theoretically, the shape of the composite structure made by 4DPC (particularly the U-structures) can be affected by this temperature fluctuation. It is important to know the magnitude of the shape change due to this temperature fluctuation for effective operation. Experimental work was carried out to determine the magnitude of the change. A theoretical calculation procedure was also developed for the prediction of this shape change. Cylindrical shells made by 4DPC were subjected

to temperature changes. The shape of these structures was determined at different temperatures.

5.7.1.1 Sample preparation

- Three samples were manufactured from carbon/epoxy prepregs (CYCOM 922) using 4DPC. The lay-up sequence was [0/90]. The samples have a length of 300 mm and a width of 150 mm.
- The three samples were cut in half (as shown in Figure 5.35) and the edges of these samples were smoothed after cutting using a file and sandpaper. The samples were labeled as samples 1A, 1B, 2A, 2B, 3A, and 3B. Each of these samples was marked with upper and lower regions for measuring the radius (Figure 5.36).

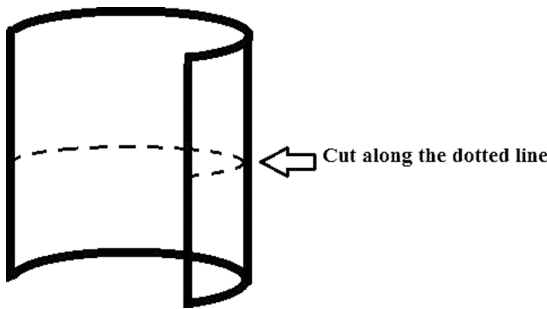


Figure 5.35: The sample was cut into halves to increase the number of replicates.



Figure 5.36: Six samples.

5.7.1.2 Conditioning of the samples

In order to isolate the effect of temperature on the change in shape of the samples, it was felt necessary to remove moisture from the samples before thermal treatment. The moisture that exists inside the samples due to exposure to the environment can be considered to consist of two types: free moisture and bound moisture. Free moisture consists of water molecules that are not bound to the molecules of the polymer, while the bound moisture consists of water molecules that are bound to the molecules of the polymer.

5.7.1.2.1 Removal of free moisture

The samples were placed inside a desiccator (containing anhydrous calcium sulfate) under vacuum to desorb the free moisture that exists within them. The removal of free moisture from the sample or material should result in a change in net strain in the material, as the removal of moisture eliminates the net stress and net moment due to the moisture. This causes the sample to have a smaller radius than the one with moisture present.

While the samples were placed inside the desiccator, they were checked for the radius (using the perpendicular bisector chord method) and weight. This process was done every 12 h for the first 5 days, and later, when it stabilized, the readings were taken every 24 h.

The moisture concentration at any particular time is determined using the following equation:

$$\Delta C = \frac{W - W_d}{W_d} \times 100 \quad (5.50)$$

where ΔC is the moisture concentration at any time, W is the weight of the sample at any time (g), and W_d is the weight of the dry sample (g). This weight is determined at the end of the drying period.

The weight of all six samples was collected over time and is plotted in Figure 5.37.

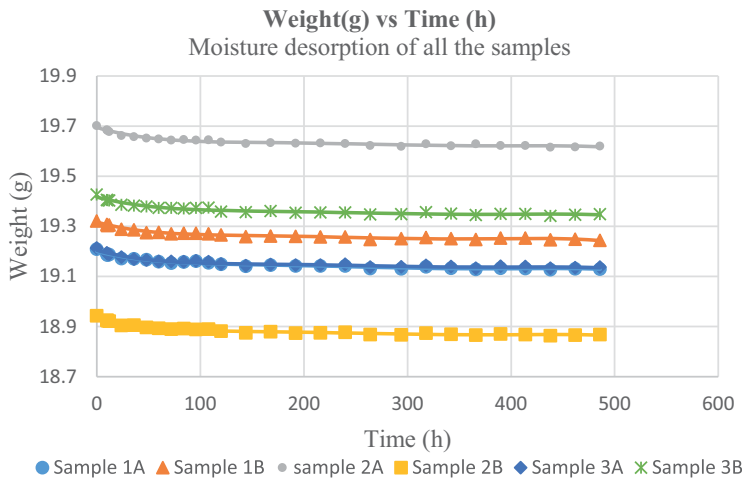


Figure 5.37: Weight of all samples over time.

From eq. (5.50), the moisture concentration as a function of time was calculated. The moisture concentrations are plotted versus the square root of time, as shown in Figure 5.38.

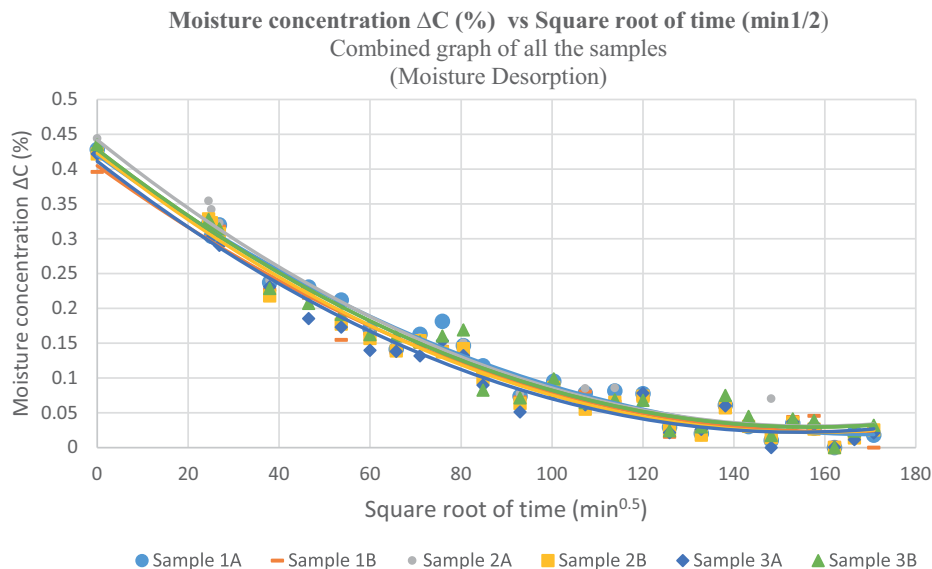


Figure 5.38: Moisture concentration as a function of the square root of time.

5.7.1.2.2 Removal of bound moisture

The bound moisture molecules have a molecular bond with the plastic molecules. In order to remove them, placing the samples inside a desiccator is not sufficient. They can be removed by heating the sample to a sufficiently high temperature.

The samples were placed in the oven at 180 °C for 20 h and then heated again for 3 h at the same temperature. This was done to confirm that there was no weight change after the initial heating. During this experiment, both the radius and weight of the samples were previously measured. This was repeated for all six samples on which the free moisture removal test was performed. Table 5.5 shows the weights of the samples before and after heating. It can be seen that there is no significant difference between the two sets of values. This shows that the amount of bound moisture is not significant.

Table 5.5: Weights of samples after removal of free moisture and after removal of bound moisture.

Sample name	Average weight (g) (after removal of free moisture)	Average weight (g) (after removal of bound moisture)
Sample 1A	19.13 ± 0.003	19.12 ± 0.003
Sample 1B	19.25 ± 0.004	19.24 ± 0.004
Sample 2A	19.62 ± 0.005	19.61 ± 0.005
Sample 2B	18.87 ± 0.003	18.86 ± 0.003
Sample 3A	19.14 ± 0.003	19.13 ± 0.003
Sample 3B	19.35 ± 0.004	19.34 ± 0.004

5.7.1.3 Variation of radius of curvature with time (during desorption inside desiccator)

The radii measured at various time intervals were plotted against time for all six samples and are shown in Figure 5.39. There is a significant reduction in the radius of curvature during the first 70 h. Then, the radius is stable. The final stable radius is given in Table 5.6.

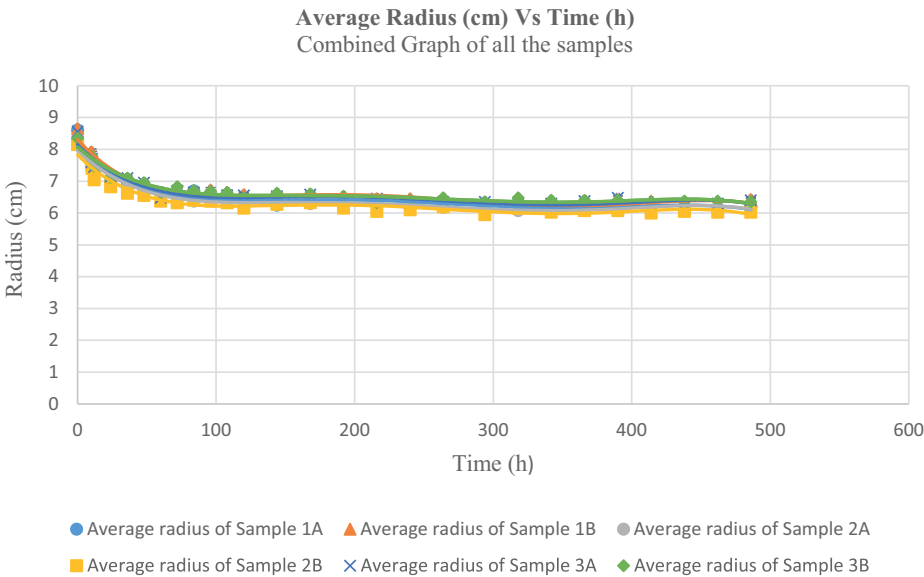


Figure 5.39: Average radius (cm) versus time (h): combined graph of all samples.

Table 5.6: Final average radius to which the samples curve up after desorption.

Sample name	Average radius (cm)
Sample 1A	6.19 ± 0.04
Sample 1B	6.31 ± 0.07
Sample 2A	6.12 ± 0.05
Sample 2B	6.03 ± 0.05
Sample 3A	6.30 ± 0.06
Sample 3B	6.36 ± 0.04

Table 5.7 gives a summary of the changes in weights and radii for all samples. It can be seen that there is a significant change in the radius of curvature due to water desorption. The average radius due to desorption of moisture decreases from 8.44 cm down to 6.08 cm (28%).

Table 5.7: Summary of changes in weights and radii of curvature.

Sample name	Average radius (cm) (before removal of moisture)	Average radius (cm) (after removal of moisture)	Weight (g) (before removal moisture)	Average weight (g) (after removal moisture)
Sample 1A	8.6 ± 0.01	6.1 ± 0.03	19.21	19.12 ± 0.003
Sample 1B	8.65 ± 0.05	6.1 ± 0.04	19.32	19.24 ± 0.004
Sample 2A	8.33 ± 0.03	6 ± 0.04	19.70	19.61 ± 0.005
Sample 2B	8.15 ± 0.1	6 ± 0.03	18.94	18.86 ± 0.003
Sample 3A	8.55 ± 0.05	6.15 ± 0.01	19.21	19.13 ± 0.003
Sample 3B	8.35 ± 0.15	6.15 ± 0.04	19.43	19.34 ± 0.004
Average	8.44	6.08	19.3	19.22

5.7.1.4 Thermal cycling

Thermal cycling is defined as the process of subjecting a material to various temperatures with sufficient dwell time at each temperature to attain thermal equilibrium. The process is conducted gradually at a slow pace to allow time to reach a certain temperature state.

5.7.1.4.1 Sample preparation

After removing both free moisture and bound moisture, two samples (1A and 3B) were chosen, and a thermal cycling experiment was performed in an oven (Figure 5.40) with the following temperature schedules:

1. Heated up from 25 °C. Pauses were taken at 40, 60, 80, 100, 120, 140, 160, 180, and 190 °C to take measurements.
2. Cooling down after heating from 190 °C. Pauses were taken at 180, 160, 140, 120, 100, 80, 60, 40, and 25 °C for measurements.
3. This is a step-by-step process. At each temperature, a dwell period of 5 min was taken to allow for thermal equilibrium and to take photographs for radius of curvature determination.
4. While placing the sample inside the oven, a ruler was placed alongside the sample. This ruler was used for radius measurement for the given scale in Digimizer (explained in Section 5.6.1.3.3).
5. During both the heating and cooling processes, photographs of the sample at each specified temperature were taken to measure the radius at that particular temperature.
6. The photographs taken were analyzed using Digimizer (an image analysis software) [3] to calculate the radius.

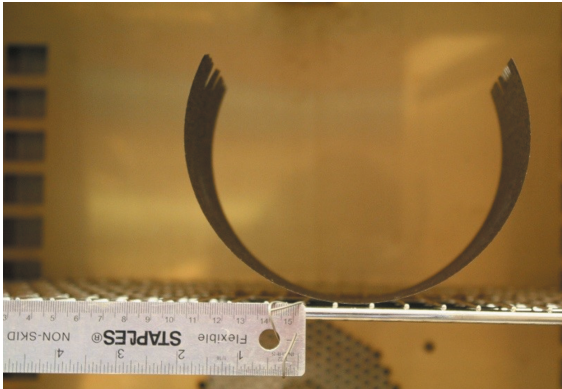


Figure 5.40: Sample placed in an oven for temperature cycling.

5.7.1.4.2 Image analysis

The image files were opened in the Digimizer software, and the scale of the image was defined using the “Unit” tool in the software. The method to set the scale of the image is by placing a ruler (as shown in Figure 5.41) beside the sample and defining a length on the ruler using “Unit” tool. In this way, Digimizer [3] calculates the number of pixels in the image with the help of the scale defined by the user (in this case, the scale is defined by marking a defined length on the ruler).

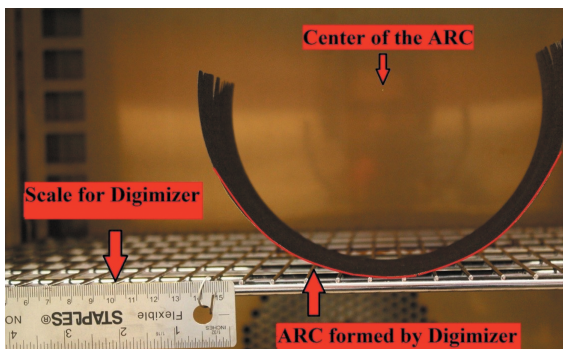


Figure 5.41: Finding the center of an arc using Digimizer [3].

Now, after defining the scale, the “center of the circular path” tool in the software is used to mark various points around the curvature of the samples in the image files. After marking the various points along the curvature of the samples, the software finds an average radius and center of the arc formed by the points around the curve.

Checking the measurement list for each file gives the value of the radius and the location of the center of the curvature.

Figure 5.42 shows the shape of the sample at different temperatures.

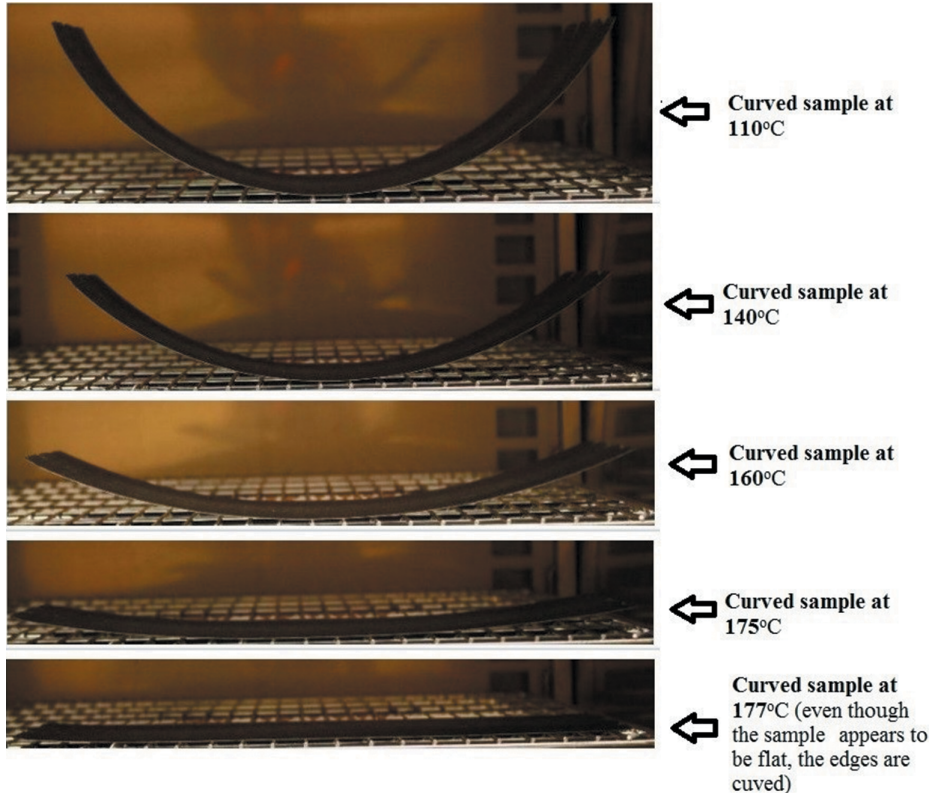


Figure 5.42: Shape of the sample at different temperatures.

5.7.1.5 Results

After analyzing the radii using Digimizer for various temperatures mentioned in the temperature cycling experiment, a graph was plotted between average radius (cm) versus temperature ($^{\circ}\text{C}$) for both sample 1A and sample 3B. The graphs plotted for heating up and cooling down processes are shown in Figure 5.43a and b. From the combined graphs for heating up and cooling down of both samples 1A and 3B, it can be seen that the material follows a nonlinear variation that can be fitted with a fifth-degree polynomial equation. The equations of both graphs are shown in the figures.

Since this test was performed after the desorption of the sample, the change in the radius of the samples is due to the effects of temperature.

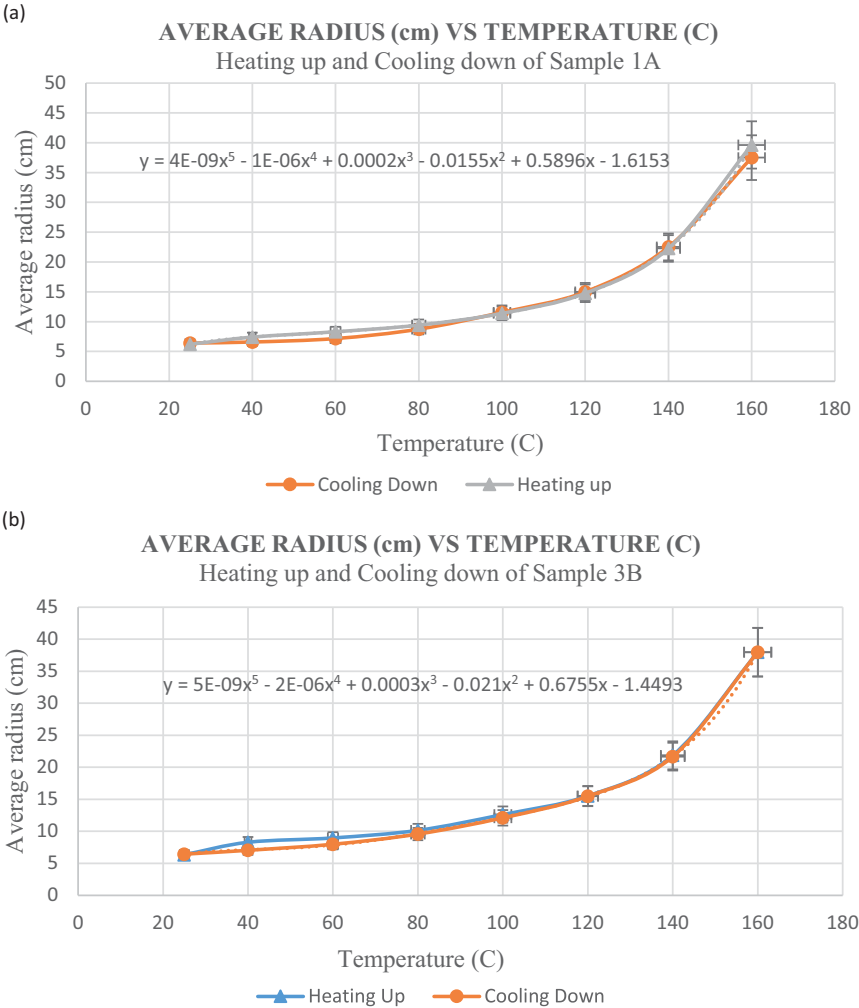


Figure 5.43: Average radius (cm) versus temperature (°C): heating up and cooling down of (a) sample 1A and (b) sample 3B.

The radii of curvature at different temperatures were also calculated using laminate theory and the material properties in Table 1.1. A comparison between the theoretical value of the radius for their respective temperatures and the experimental radius obtained using laminate theory is shown in Figure 5.44.

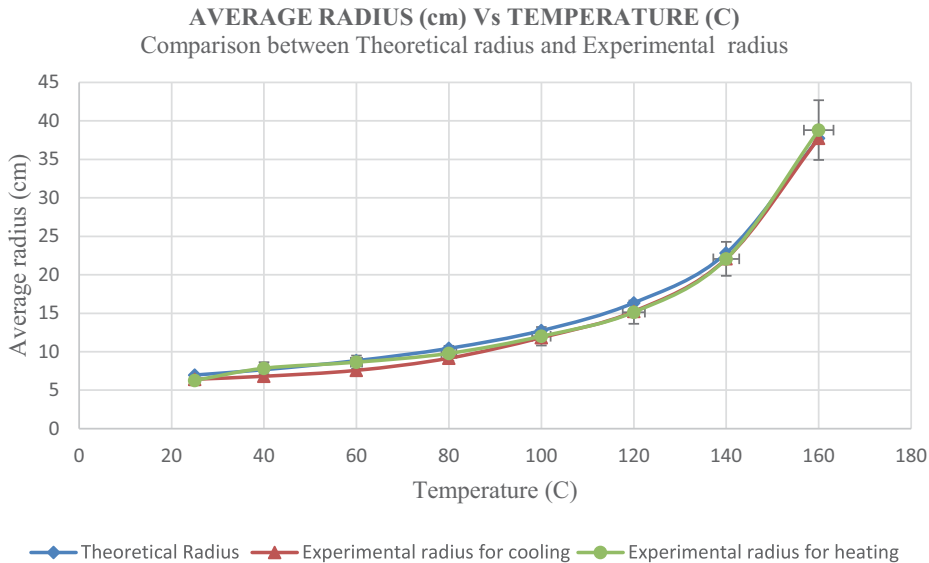


Figure 5.44: Average radius (cm) versus temperature (°C). Comparison between the theoretical radius and experimental radius.

5.7.1.5 Discussion

- From Figure 5.44, the experimental and theoretical radius values agree with each other. The variation of radius for changes in temperature is a curve of a fifth-degree polynomial equation. The curves show linear proportionality at lower temperatures and nonlinearity at higher temperatures.
- For the range of temperatures from room temperature of 20–50 °C, the change in the radius of curvature is between 7 and 8 cm. Whether this is significant depends on the requirements of the operating conditions.

5.7.2 Effect of moisture absorption on radius of curvature

When the composite material is exposed to its surroundings after curing, the sample tends to absorb moisture through diffusion from the surrounding atmosphere. This process is mainly dependent on the diffusivity of the moisture into the material.

When moisture diffuses into the samples, there is a change in the net weight of the samples, and swelling occurs. This leads to a change in the radius of the sample.

Moisture diffusion into the resin leads to a reduction of glass transition temperature and softening of the resin system, which results in degradation of stiffness and strength. The moisture enters locations where there are loose ends inside the cured composite. Loose ends inside the composites are the locations where the samples are

not entirely cured. Once the moisture enters this region, it may swell and plasticize that region. Diffusion of moisture to a condition of spatial uniformity can take months, even years, depending on the thickness of the laminate.

To understand the effect of moisture in composite materials, a study of the absorption of moisture in composite samples made using 4DPC was performed.

5.6.2.1 Sample preparation

Two samples of carbon epoxy (CYCOM 922) material with the lay-up sequence and dimensions shown in Figure 5.45 were manufactured using 4DPC. The samples were labeled as sample 1 and sample 2 along with marking the upper and lower surfaces (as shown in Figure 5.46) for measuring the radius and the weight of the samples.

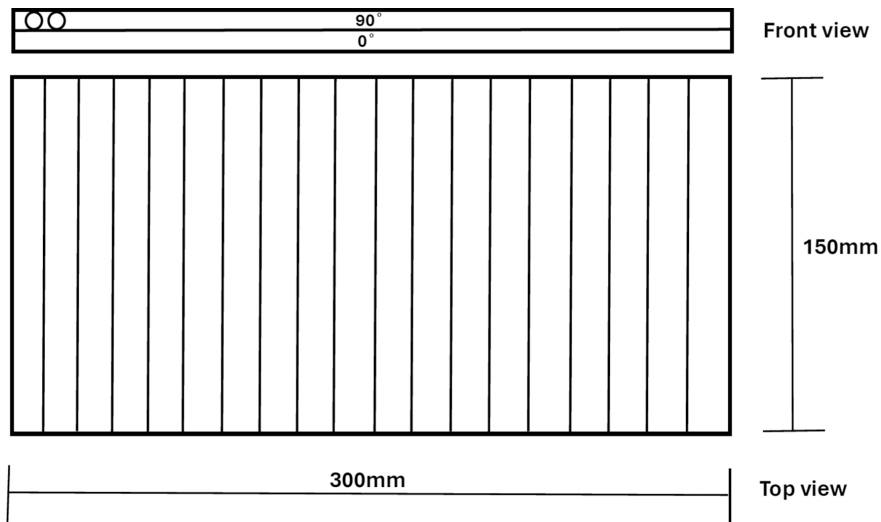


Figure 5.45: Lay-up sequence of material for moisture absorption.

The samples were stored in a room under ambient conditions. A humidity and temperature logger was used to record the relative humidity and temperature of the room for every 15 min during the whole course of the experiment. The range of relative humidity and temperature changes is shown in Figure 5.47. It is observed that the temperature at which the samples were placed was fairly constant, and the relative humidity change was not very significant. This variation in relative humidity does not have a large impact on the change in radius, as diffusion is a very slow process.

The radius and weight measurements of dry samples were recorded. Both the weight and radius were recorded at time intervals of every 1 h for the first 5 h, then every 5 h until 53 h of exposure to surroundings. Later, the readings were taken every

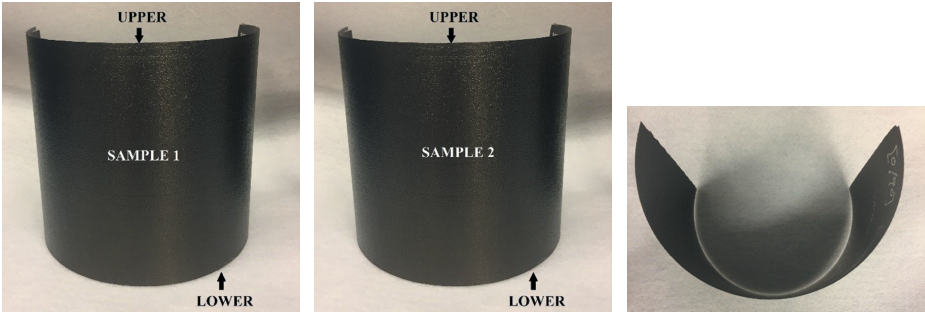


Figure 5.46: Moisture absorption samples.

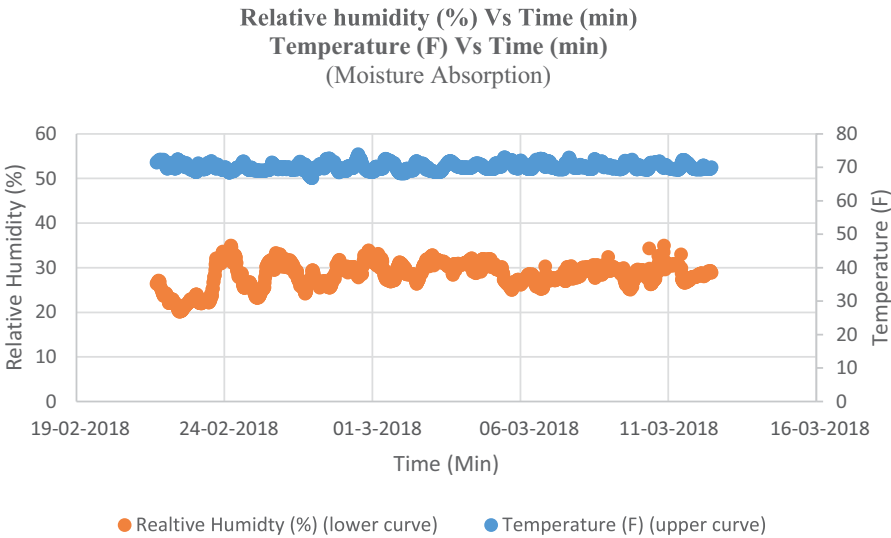


Figure 5.47: Temperature and relative humidity in the room.

12 h until 161 h. of total time after curing. Eventually, this time interval between the readings was increased to 24 h until the weight of the sample attained a constant weight. The weights of the sample were measured using a weighing scale with an accuracy of 0.1 mg.

5.6.2.2 Weight increase

Figures 5.48–5.52 show the weight increase as a function of time.

From Figures 5.48 and 5.49, it is clear that the rate of moisture absorption for the first 0–100 h is high, and later on, the moisture absorption reaches an asymptotic value. To observe the diffusion of the moisture into the samples, the moisture content

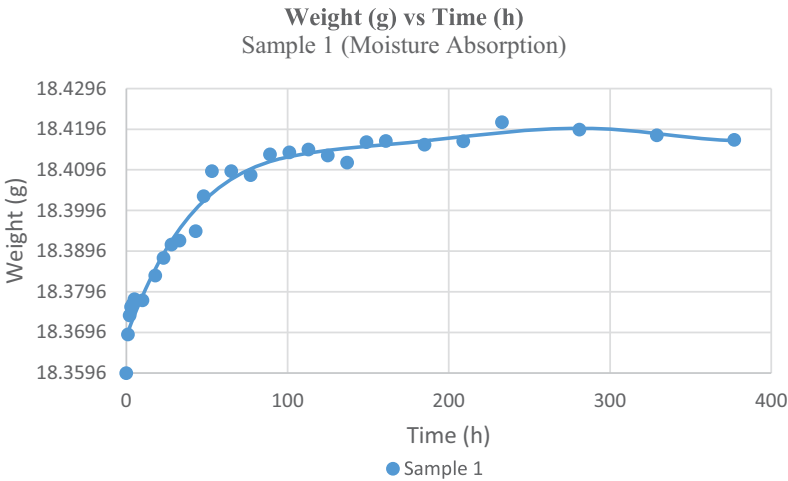


Figure 5.48: Weight (g) versus time (h) for sample 1 during moisture absorption.

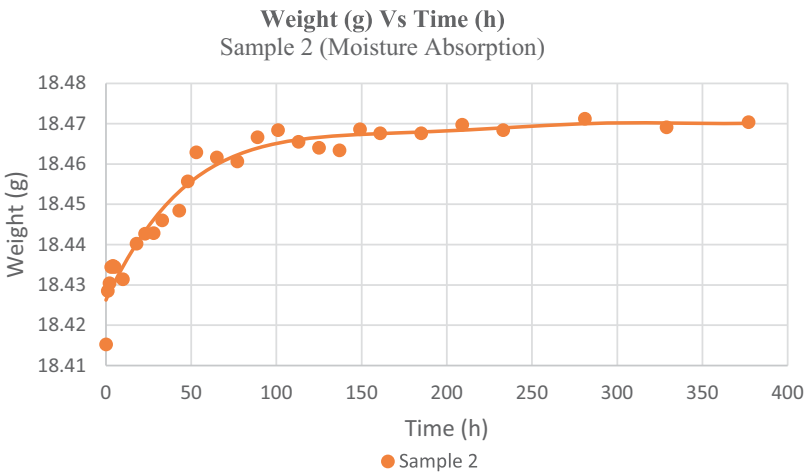


Figure 5.49: Weight (g) versus time (h) for sample 2 during moisture absorption.

(ΔC) (as shown in eq. (5.50)) is plotted against the square root of time, as shown in Figures 5.50 and 5.51.

Figure 5.52 shows the summary results from the two samples.

It can be seen that there is some variation in the maximum moisture content between sample 1 and sample 2, in spite of the fact that these samples were taken from the same batch of prepregs and made the same way. This difference is due to the variability inherent in composite materials.

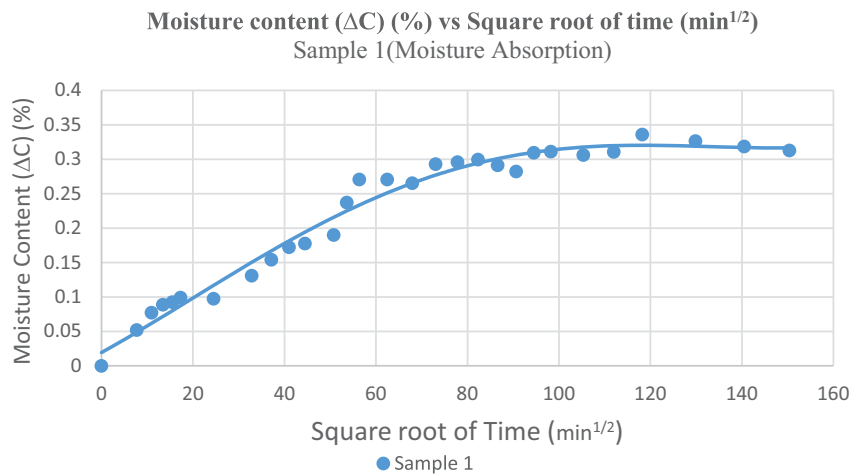


Figure 5.50: Moisture content (ΔC) (%) versus square root of time ($\text{min}^{1/2}$) for sample 1 (moisture absorption).

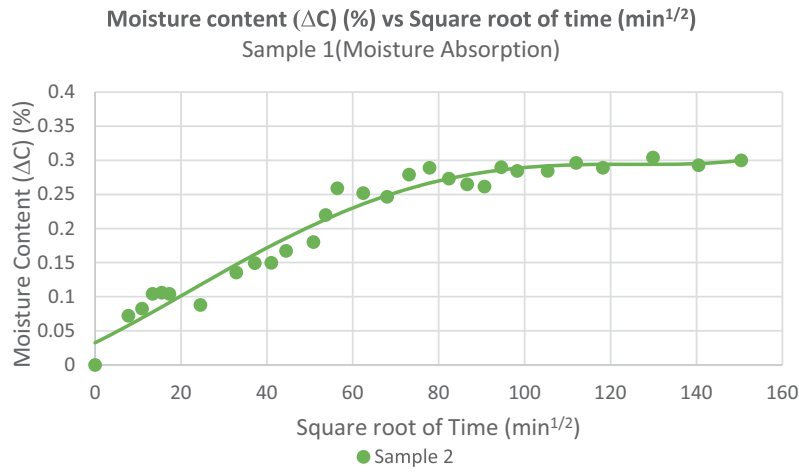


Figure 5.51: Moisture content (ΔC) (%) versus square root of time ($\text{min}^{1/2}$) for sample 2 (moisture absorption).

5.6.2.3 Change in radius of curvature

Figures 5.53 and 5.54 show the change in the radius of curvature as a function of time. From Figures 5.53 and 5.54, it is clear that there is a significant change in radius for the first 0–100 h. This is the period during which the rate of moisture diffusion into the material is at its maximum, and this can be validated using a weight (g) ver-

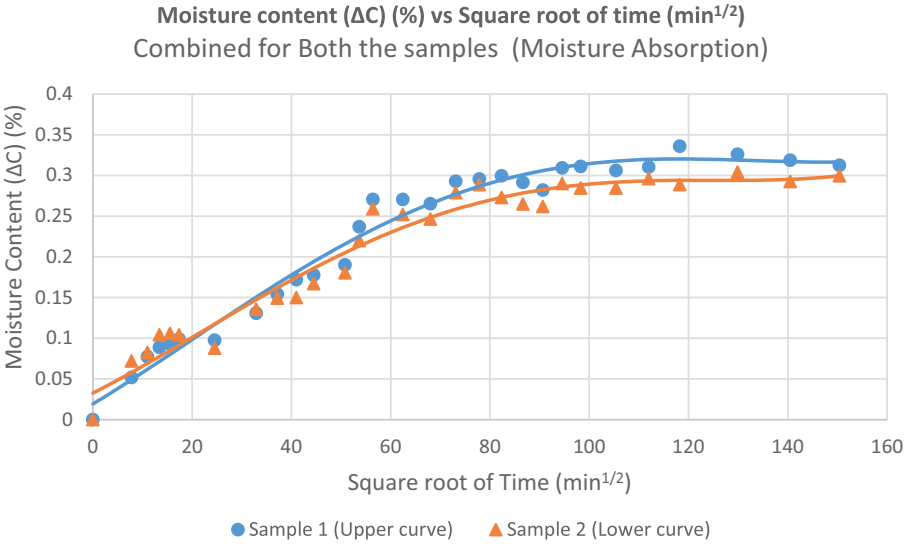


Figure 5.52: Moisture content (ΔC) (%) versus square root of time ($\text{min}^{1/2}$) for both samples.

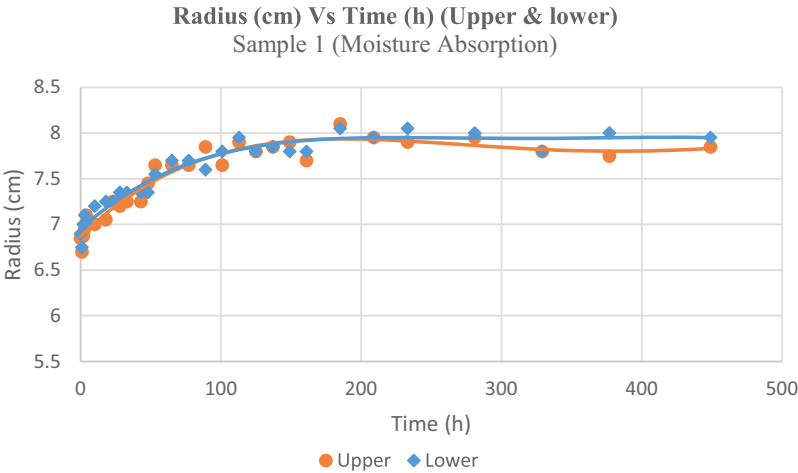


Figure 5.53: Radius (cm) versus time (h) (upper and lower): Sample 1.

sus time (h) graph shown in Figure 5.48. The final average radius of the sample obtained at the end of the experiment is given in Table 5.8.

The difference between the values of the radius (Table 5.8) is due to the variability inherent in composite materials.

Data from Figure 5.52 are cross-plotted with data from Figures 5.53 and 5.54 to obtain the variation in the radius of curvature with the change in moisture content,

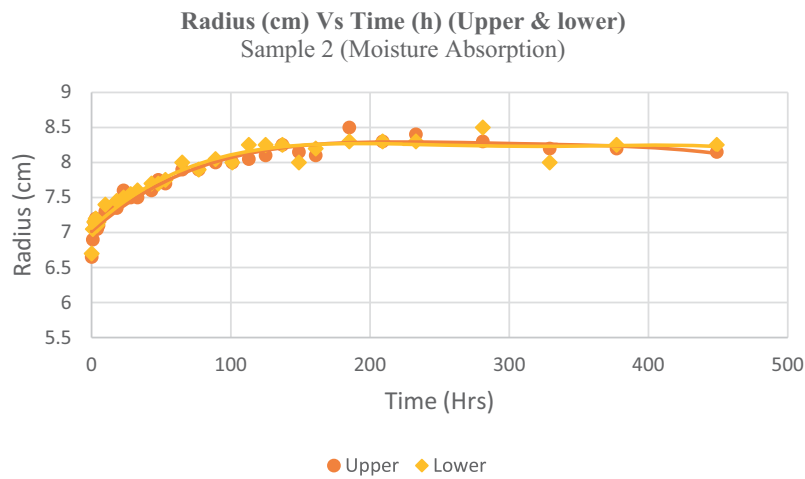


Figure 5.54: Radius (cm) versus time (h) (upper and lower): Sample 2.

Table 5.8: Final radius of the sample after moisture absorption.

Samples name	Final average radius (cm)
Sample 1	7.94 ± 0.16
Sample 2	8.28 ± 0.20

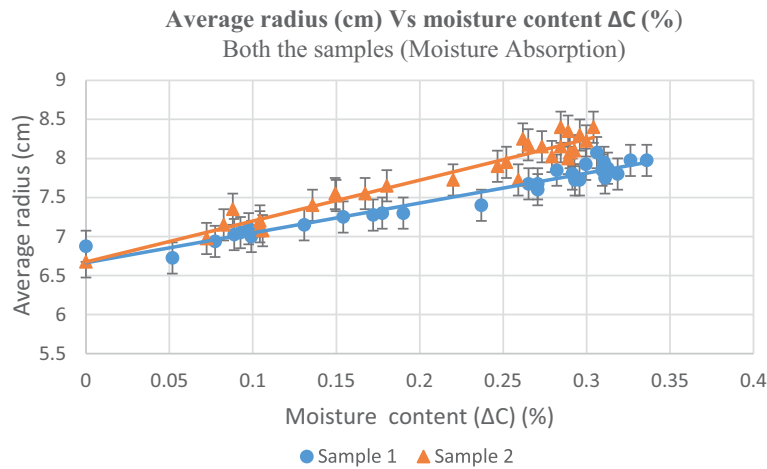


Figure 5.55: Average radius (cm) versus moisture content ΔC (%) for both samples (moisture absorption).

as shown in Figure 5.55. From these graphs, it can be seen that the radius of curvature is linearly proportional to the moisture content.

Appendix

MATLAB code for unsymmetric laminates

```
clearvars;
clc;
%Set the variables and parameters
E1=155e9;E2=12.16e9;G12=4.4e9;v12=0.248;
a1=-0.018e-6;a2=24.3e-6; %alpha
theta1=0;theta2=pi/4;
h=1.25e-4; %thickness in m
dT=-157;
v21 = v12*E2/E1;

% in GPa
d=(1-v12*v21);
q11=E1/d;
q12=v21*E1/d;
q22=E2/d;
q66 = G12;

theta=[theta1 theta2];
% for layers
for i=1:2
    m=cos(theta(i)); n=sin(theta(i));
    if abs(m) < 1e-7
        m=0;
    end
    if abs(n) < 1e-7
        n=0;
    end
    Q11(i) = q11*m^4 + (q12 + 2*q66)*n^2*m^2 + q22*n^4;
    Q12(i) = (q11 + q22 - 4*q66)*n^2*m^2 + q12*(n^4 + m^4);
    Q16(i) = (q11 - q12 - 2*q66)*n*m^3 + (q12 - q22 + 2*q66)*n^3*m;
    Q22(i) = q11*n^4 + 2*(q12 + 2*q66)*n^2*m^2 + q22*m^4;
    Q26(i) = (q11 - q12 - 2*q66)*n^3*m + (q12 - q22 + 2*q66)*n*m^3;
    Q66(i) = (q11 + q22 - 2*q12 - 2*q66)*n^2*m^2 + q66*(n^4 + m^4);
    ax(i) = a1*m^2 + a2*n^2;
```

```

ay(i) = a1*n^2 + a2*m^2;
axy(i) = 2* (a1 - a2)*m*n;
end

```

```

A11 = h*sum(Q11);
A12 = h*sum(Q12);
A16 = h*sum(Q16);
A22 = h*sum(Q22);
A26 = h*sum(Q26);
A66 = h*sum(Q66);

```

```

B11 = h^2/2 * (-Q11(1) + Q11(2));
B12 = h^2/2 * (-Q12(1) + Q12(2));
B16 = h^2/2 * (-Q16(1) + Q16(2));
B22 = h^2/2 * (-Q22(1) + Q22(2));
B26 = h^2/2 * (-Q26(1) + Q26(2));
B66 = h^2/2 * (-Q66(1) + Q66(2));

```

```

D11 = h^3/3 * sum(Q11);
D12 = h^3/3 * sum(Q12);
D16 = h^3/3 * sum(Q16);
D22 = h^3/3 * sum(Q22);
D26 = h^3/3 * sum(Q26);
D66 = h^3/3 * sum(Q66);

```

%Thermal load and moment resultants

```

NTx = h*dT*(Q11(1)*ax(1) + Q12(1)*ay(1) + Q16(1)*axy(1));
NTx = NTx + h*dT*(Q11(2)*ax(2) + Q12(2)*ay(2) + Q16(2)*axy(2));
NTy = h*dT*(Q12(1)*ax(1) + Q22(1)*ay(1) + Q26(1)*axy(1));
NTy = NTy + h*dT*(Q12(2)*ax(2) + Q22(2)*ay(2) + Q26(2)*axy(2));
NTxy = h*dT*(Q16(1)*ax(1) + Q26(1)*ay(1) + Q66(1)*axy(1));
NTxy = NTxy + h*dT*(Q16(2)*ax(2) + Q26(2)*ay(2) + Q66(2)*axy(2));
MTx = h^2/2*dT*(-(Q11(1)*ax(1)) + Q12(1)*ay(1) + Q16(1)*axy(1));
MTx = MTx + h^2/2*dT*(Q11(2)*ax(2) + Q12(2)*ay(2) + Q16(2)*axy(2));
MTy = h^2/2*dT*(-(Q12(1)*ax(1)) + Q22(1)*ay(1) + Q26(1)*axy(1));
MTy = MTy + h^2/2*dT*(Q11(2)*ax(2) + Q22(2)*ay(2) + Q26(2)*axy(2));
MTxy = h^2/2*dT*(-Q16(1)*ax(1) - Q26(1)*ay(1) - Q66(1)*axy(1));
MTxy = MTxy + h^2/2*dT*(Q16(2)*ax(2) + Q26(2)*ay(2) + Q66(2)*axy(2));

```

% Inversion by partition

```
AM = [A11 A12 A16; A12 A22 A26; A16 A26 A66];
```

```
BM = [B11 B12 B16; B12 B22 B26; B16 B26 B66];
```

```
DM = [D11 D12 D16; D12 D22 D26; D16 D26 D66];
```

```
NM = [NTx NTy NTxy];
```

```
MM = [MTx MTy MTxy];
```

```
AI = inv(AM);
```

```
BI = inv(BM);
```

```
DI = inv(DM);
```

```
MA = AI*BM - BI*DM;
```

```
MAI = inv(MA);
```

```
FT1 = MAI*AI;
```

```
FT2 = FT1*NM';
```

```
ST1 = MAI*BI;
```

```
ST2 = ST1*MM';
```

```
K = FT2 - ST2;
```

References

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Chapter 6

C-structures

C-structures are composite structures made using the technique of 4D printing of composites (4DPCs). In these applications, the structures have their edges fixed either to other structures or to themselves. The manufacturing will consist of two steps. The first step is the manufacturing of a structure with some free edges using 4DPC. The second step is the fixing of the free edge by joining it to another part of the same structure or joining it to another structure. Joining can be bonding, bolting, or some other means. Two cases will be presented in this chapter. The first is the composite omega stiffener. This structure has two parts: one flat base at the bottom and one curved skin at the top. These two are bonded to each other. As such, they provide constraint to each other. The second structure is the corrugated core for flexible wings. The core is bonded to the upper skin and the lower skin of the wing. The skins provide constraint to the corrugated core.

6.1 Composite omega stiffener

The following presents the manufacturing procedure for the composite omega stiffener. A more complete presentation, including the mechanical performance of the stiffener, can be found in reference [1].

6.1.1 General

In aircraft structures, stiffeners are usually used to reinforce the thin shells used in wings or the aircraft body. The stiffeners can have many shapes, as shown in Figure 6.1. Among the different shapes, the “hat” or “omega” stiffener is of particular interest since it presents challenges to the method of 4DPC.

The “hat” in the omega stiffener can either have sharp corners, as shown in Figure 6.1, or it can be curved, as shown in Figure 6.2. The dimensions shown in Figure 6.2 are used for the construction of samples using 4DPC.

Different features in Figure 6.2 that must be taken into consideration for 4DPC are:

- There is a flat base and a curved upper skin. These are bonded at both the left and right edges. There is empty space between the flat base and the curved upper skin. It is not possible to provide bonding at both the left and right edges in the flat configuration. Bonding at both the left and right edges in the flat configuration will prevent the separation between the lower base and the upper skin.
- The upper skin has a portion that is concave upward and another portion that is concave downward, with an inflection in between. This can be made using the experience gained in making the S-shaped structures in Section 5.1.

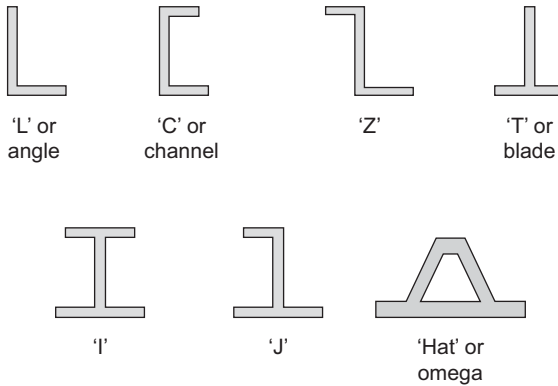


Figure 6.1: Different shapes for stiffeners [1].

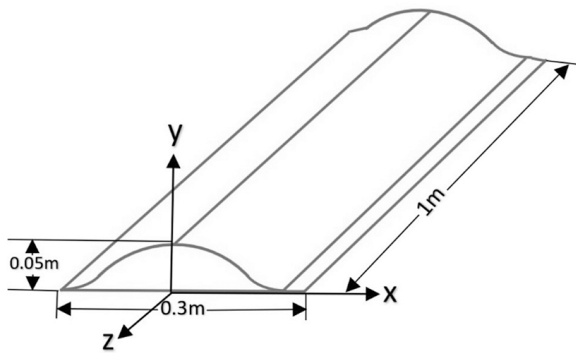


Figure 6.2: Omega stiffener with curved “hat” [1].

6.1.2 Lay-up sequence

The lay-up sequence is as shown in Figure 6.3.

This lay-up sequence has the following features:

- Base plate:** The base plate serves as the flat (lower) part of the stiffener. It has a width of 0.3 m. The lay-up sequence is $[0/90]_{4s}$. It has a total of 16 layers and a thickness of 2 mm.
- Brass shim:** On top of the base plate is a thin layer of brass shim. It covers the entire width of the base plate except for a distance of 0.031 m. The brass shim serves to allow the top part of the stiffener to separate from the base plate after curing and cooling to room temperature. The brass shim does not cover the 0.031 m region to allow bonding to take place.
- Layer above the brass shim:** The layer above the brass shim has a few segments. The leftmost segment has a length of 0.082 m. It has the lay-up $[0]_1$. The left por-

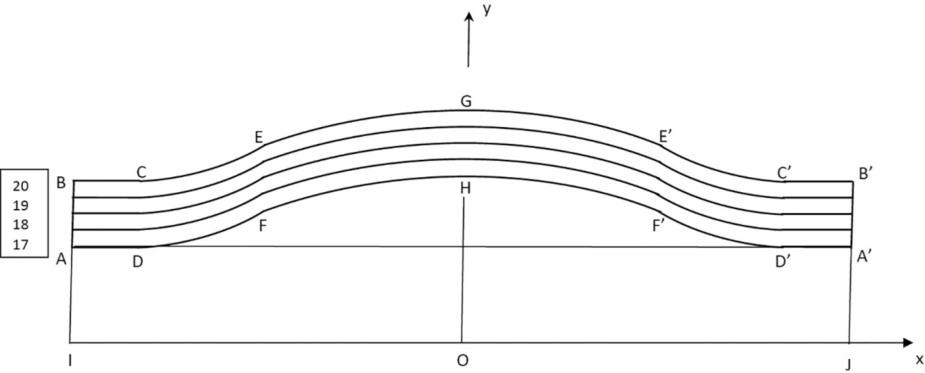


Figure 6.4: Schematic of the arrangement.

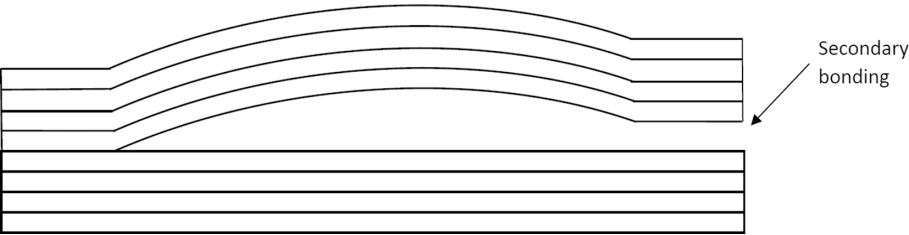


Figure 6.5: Stiffener after curing and cooling to room temperature.

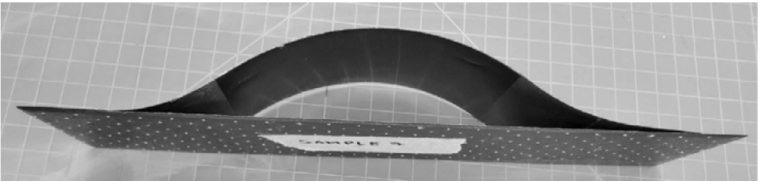


Figure 6.6: Final omega stiffener.

6.1.3 Discussion

The making of the omega stiffener using 4DPC illustrates that it is possible to make composite structures with openings using this method. Only thin structures can be made. As such, the application may be limited to small airplanes.

6.2 Corrugated cores for sandwich constructions

6.2.1 Introduction

The concept of the S-shaped construction in Section 5.1 can be extended to make corrugated structures. This can be done by extending the laminate to contain more than one joint. Figure 6.7 shows the concept. In Figure 6.7a, the laminate contains a continuous layer of $[0]_n$ lay-up. There are short strips of $[90]_m$ that are placed on top and bottom of the $[0]_n$ layer. Figure 6.7b shows the resulting deformed configuration for the case $m = n$. The curvature of each lobe depends on the values of m and n .

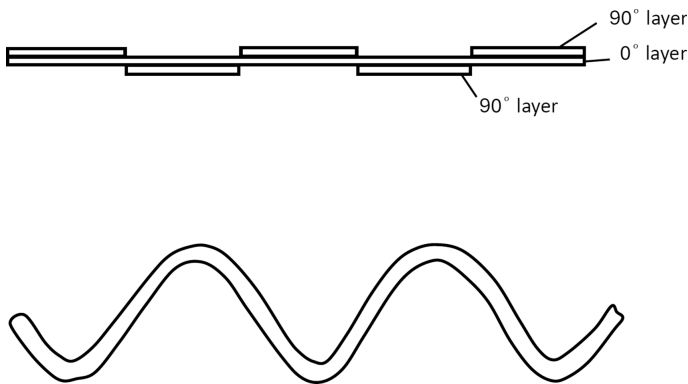


Figure 6.7: Corrugation with uniform amplitude and no overlap between 90° segments.

The arrangement as shown in Figure 6.7 can have a lot of variations. One variation of the corrugated arrangement is shown in Figure 6.8. In this case, there are overlaps at the ends of the upper $[90]_m$ layer and the lower $[90]_m$ layers. It was shown in reference [2] that the amount of the overlap has an effect on the radius of curvature on both sides of the overlap. The larger the portion of the overlap relative to the total length, the larger the radius of curvature will be.

Rather than a corrugation with uniform amplitude as shown in Figure 6.7, the corrugation can be tapered as shown in Figure 6.9 (bottom). This can be done by varying the length of the $[90]_m$ segments along the length of the $[0]_n$ layer, as shown in Figure 6.9 (top).

Since the length of the overlap, as shown in Figure 6.8a, influences the radius of curvature on both sides of the overlap, the tapering can also be done by varying the length of the overlaps.

In the sandwich construction, the core needs to be bonded to an upper skin and a lower skin. The bonding is a secondary process after the core has been made using 4DPC. Figure 6.10 shows the case of a sandwich structure with constant thickness.

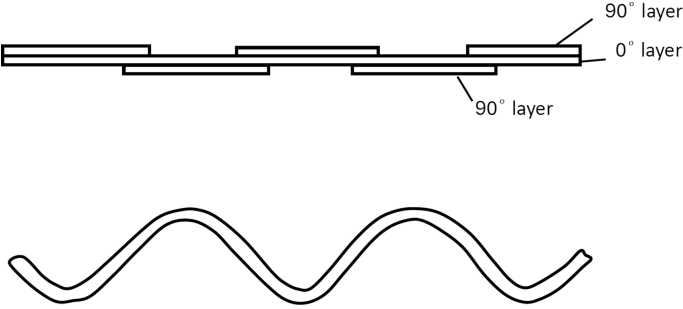


Figure 6.8: Corrugation with overlap between 90° segments.

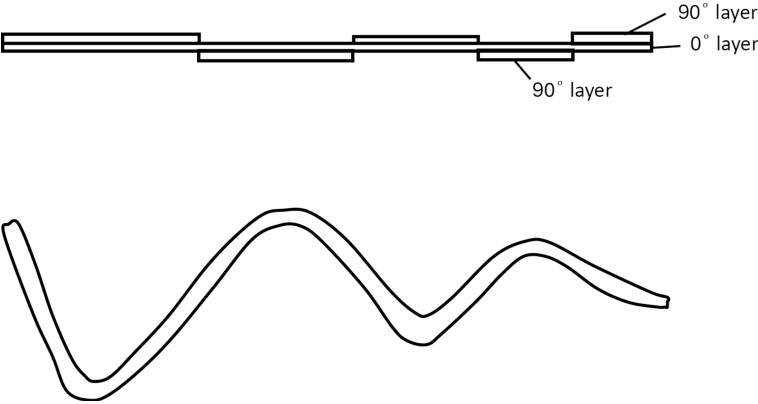


Figure 6.9: Corrugation with taper.

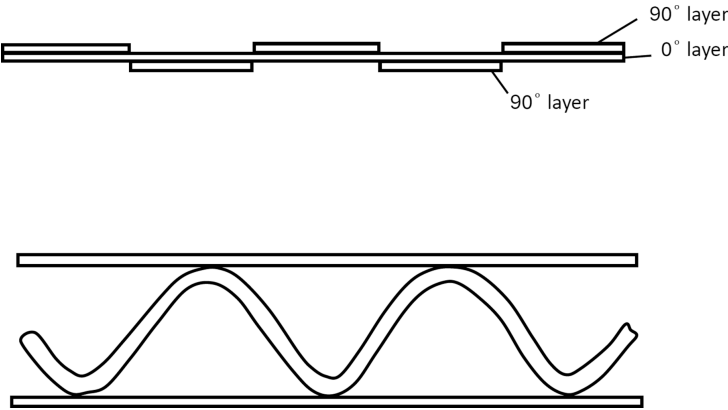


Figure 6.10: Sandwich structure with constant thickness.

6.2.2 Application example

The following presents the procedure for the construction of a part of a flexible wing using a corrugated core made by 4DPC. A more complete presentation, including the mechanical performance of the flexible wing with a corrugated composite core made by 4DPC, can be found in reference [2].

One application for the corrugated core sandwich composite structure is the trailing edge of a flexible aircraft wing, as shown in Figure 6.11.

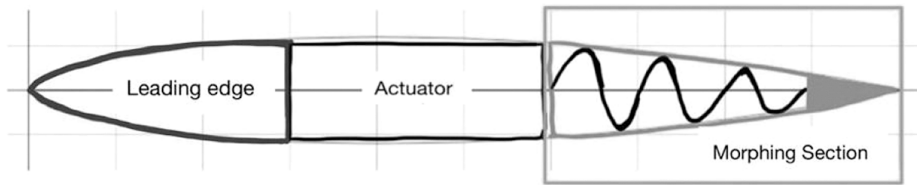


Figure 6.11: Corrugated core in the trailing edge of a flexible wing.

Figure 6.12 shows a prototype of the composite trailing edge, with a corrugated core, upper skin, and lower skin.



Figure 6.12: A prototype of the trailing edge of a flexible wing [2].

The corrugated composite core was made by the method of 4DPC using the lay-up sequence as shown in Figure 6.13.

Inserts were made to support the ends of the tapered structures and to provide connection points for loading. The final assembly with the compressed core is shown in Figure 6.14.

The prototype shown in Figure 6.14 was subjected to static cantilever loading. The maximum load was 35.5 N (7.98 lbs). When this load is converted to an equivalent distributed pressure, the value is 51.7 lbs/ft² [2]. This load falls within the range of loadings in small to medium-sized unmanned aerial vehicles [3, 4].

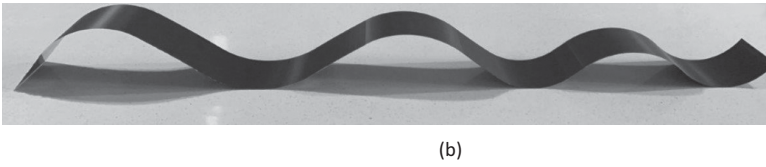
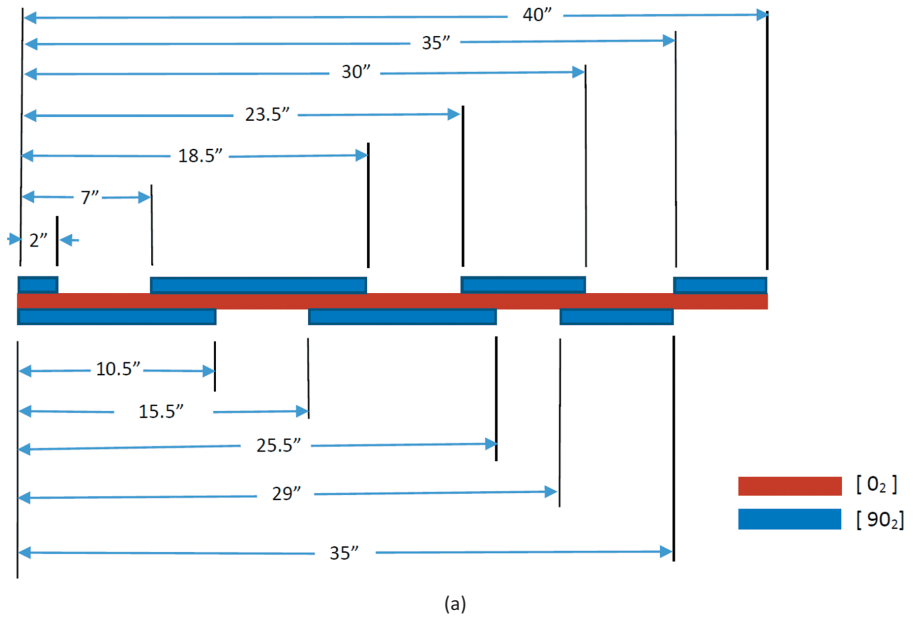


Figure 6.13: Tapered corrugation [2].



Figure 6.14: Photo of the structure.

6.2.3 Conclusion

The results from the work presented above show that it is possible to manufacture a flexible, morphable wing using a corrugated core made with the method of 4DPCs. The prototype made and tested shows that it can support a load that is within the range of loads for practical small and medium unmanned aerial vehicles.

References

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Chapter 7

A-structures

7.1 General aspects

One particular type of C-structures is the A-structures. These are structures made by 4D printing of composites (4DPCs), but their edges are constrained by bonding. They also have shapes of conventional structures such as circular cross-sectional cylinders or circular cross-sectional cones. However, if they are made using 4DPC only, their edges are not constrained. When these edges are left free, they are just circular cross-sectional shells or circular conical shell sections. When the edges are bonded together, the shells become circular cylinders or circular cones. Their cross sections deviate very slightly from that of perfect cylinders. Since there is an overlap of these edges and the edges are bonded together, these structures are termed A-structures, such as A-cylinder or A-cone. The word “A” stands for “almost.”

7.2 A-circular cylinders

A-circular cylinders are made by bonding the edges of the circular cylindrical shells made using 4DPC. These circular cylindrical shells can be made using laminates with lay-up sequences of the type $[0_m/90_n]$. A figure of this type of shell is shown in Figure 1.8. In the making of the A-circular cylinders, there are two radii that need to be considered. The initial radius R_i is determined from the lay-up sequence, and the final radius R_f is determined from the diameter of the cylinder to be made. In the 4DPC technique, one starts with a blank with length L and width b , as shown in Figure 7.1.

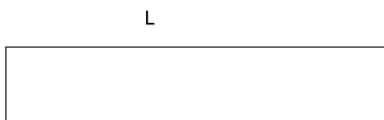


Figure 7.1: Flat blank for the making of the A-cylinder using 4DPC.

After the 4DPC process, the length L will curl up into a shell with radius R_i . The radius R_i is determined by the lay-up sequence of the laminate. As such, R_i for a laminate with a lay-up sequence $[0/90]$ would be different from R_i for a laminate with a lay-up sequence $[0_2/90]$. For situations far away from the bifurcation points, the value of R_i may be approximated using laminate theory. When the length L is short, the shell is an open shell and is not a complete circular shell.

To have a complete circular shell, the length L has to have the value of L_i which is given as follows:

$$L_i = 2\pi R_i \quad (7.1)$$

When $L < L_i$, an open circular shell is obtained (Figure 7.2a). When $L > L_i$, there is an overlap at the edges (Figure 7.2b).

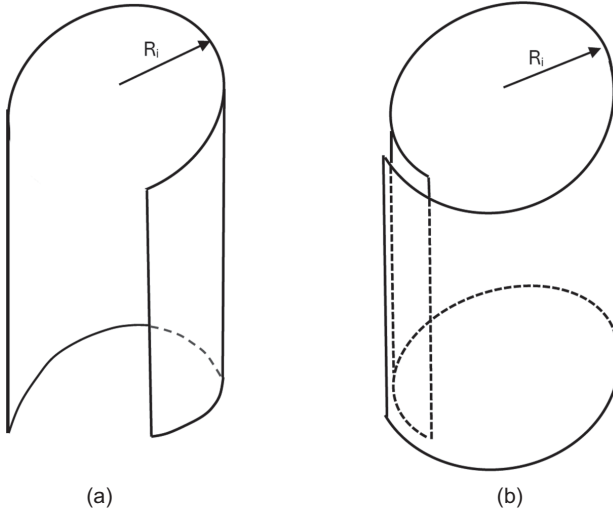


Figure 7.2: Different types of circular shells: (a) open shell and (b) A-cylinder with overlap.

One can also make a complete A-cylinder from the open shell, as shown in Figure 7.2a. This is done by forcing the two edges of the shell to overlap each other. This will become like Figure 7.2b but with a radius R_f given by the relation

$$L = 2\pi R_f + \text{overlap} \quad (7.2)$$

Note that for the case of $L > L_i$, $R_f = R_i$, but for the case of $L < L_i$, $R_f < R_i$.

When the two sides that overlap at the edges of the shell are bonded to each other, one forms an A-cylinder. Care should be taken when bonding the edges together. The clamps should have a shape that conforms to the curvature of the cylindrical shell. The clamps can be made using 3D printing. Figure 7.3 shows the clamping of the two edges with an adapter that conforms to the shape of the shell.

Since the A-cylinder has bonded edges, it is necessary to ensure that it has good circularity. This can be done by scanning the shape of the profile using a laser profilometer. Figure 7.4 shows a shape profile of an A-cylinder made using the lay-up sequence [0/90], with a length $L = 500$ mm. This is made using carbon/epoxy composite prepregs from Rock West Composites that were cured at 135 °C. Their properties are different from those in Table 1.1. The radius after curing and cooling to room temperature is 7.16 mm.

Figure 7.4 shows that there is a very small difference between the actual profile and the profile of a perfect circle. The standard deviation is 0.29%.



Figure 7.3: Clamping with a conforming adapter for bonding the edges.

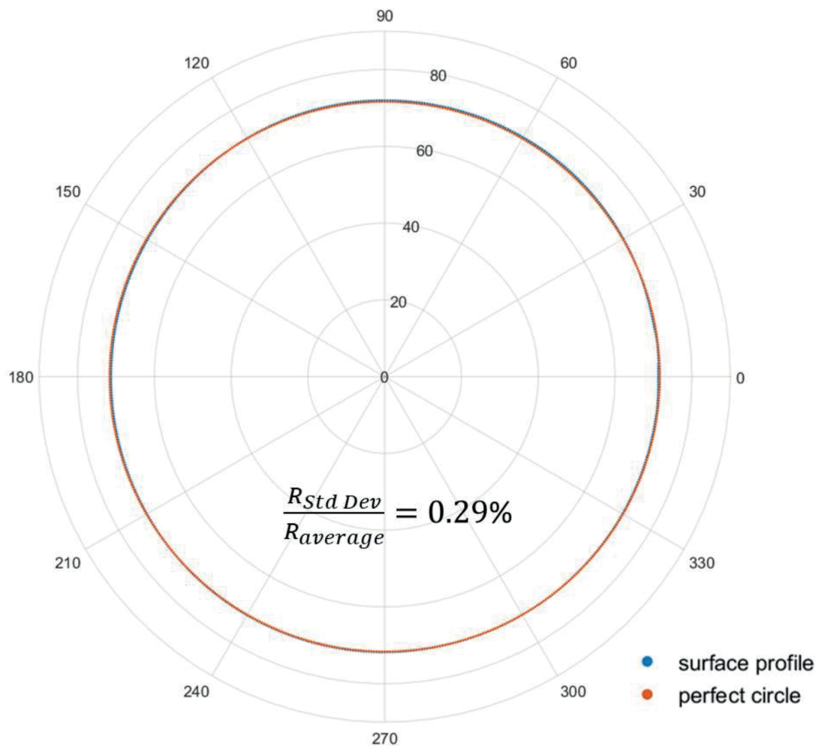


Figure 7.4: Scanned profile of an A-cylinder made from $[0_2/90]$ lay-up sequence and a length of 500 mm.

It is of interest to determine whether the A-cylinder can withstand practical axial buckling loads that are imposed on a regular cylinder.

The A-cylinder was mounted onto two supports at the ends, as shown in Figure 7.5.

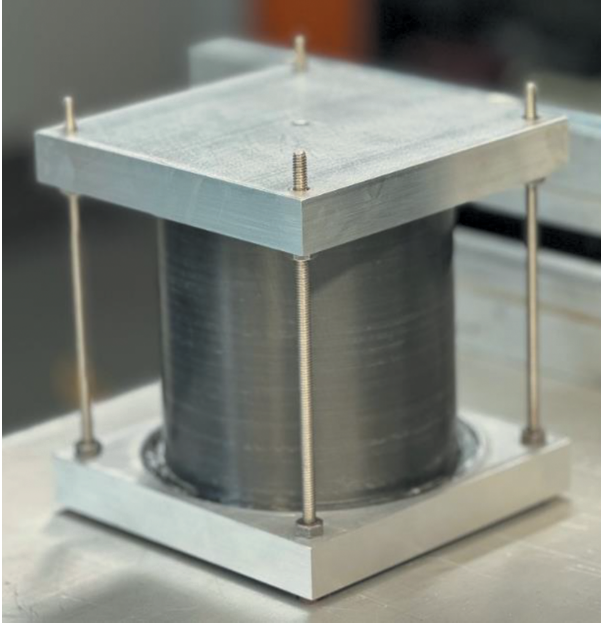


Figure 7.5: A-cylinder mounted onto two end supports.

The structure was tested in compression in an MTS testing machine. The load–displacement curve is shown in Figure 7.6. The maximum buckling load is 13,770 N. In Figure 7.6, the blue curve shows the first loading. It reaches a maximum buckling load of 13,770 N when buckling occurred. Then the load dropped to 6,000 N and stabilized. Diamond-shaped areas on the surface of the cylinder were observed, as shown in Figure 7.7. The machine was allowed to exert a few fractions of a millimeter after buckling and before unloading. When the load reached 0, there was a permanent deformation of 0.3 mm. The shape of the cylinder was fully recovered. The second loading was applied. The red line curve shows the second loading. It reached a maximum of 12,600 N when buckling occurred again. The load dropped down to 5,900 N and stabilized. The machine was allowed to exert about 2 mm more in displacement before unloading. The permanent deformation stayed at about 0.3 mm. The shape of the cylinder was fully recovered.

Figure 7.7 shows the mode of deformation. Many dimples of diamond shape appear on the surface in regions where there is no overlap. There were no dimples in the overlap region.

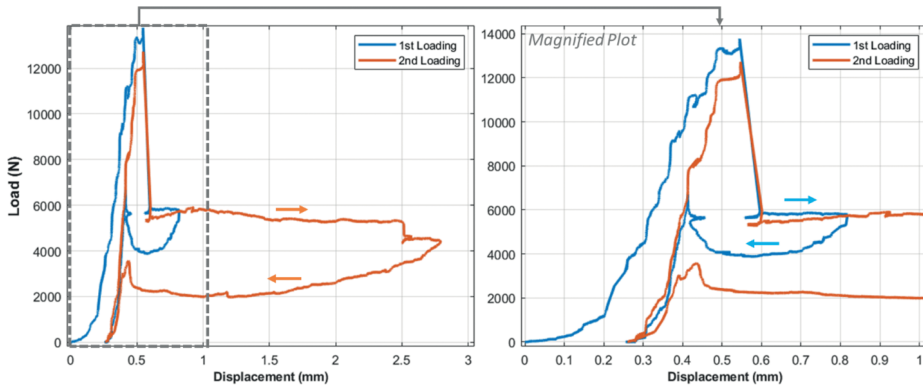


Figure 7.6: Load-displacement curve for an A-cylinder made of lay-up sequence $[0/90]$; radius is 71.6 mm and height H is 135 mm [1].



Figure 7.7: Mode of deformation of the A-cylinder [1].

7.2.1 Comparison with other tubes of equivalent weight

7.2.1.1 Aluminum cylinder

It would be of interest to compare the buckling load of the A-cylinder with that of an aluminum cylinder with the same weight. The composite A-cylinder has a weight of 54 g and a thickness of 0.386 mm. An aluminum cylinder with the same height and diameter (height of 135 mm and diameter of 143.2 mm) would have a thickness determined as follows:

$$m = \rho_1 H_1 \pi D_1 t_1 = \rho_2 H_2 \pi D_2 t_2 \quad (7.3)$$

Since the height and diameter are the same:

$$t_2 = t_1 \frac{\rho_1}{\rho_2} \quad (7.4)$$

Taking the specific gravity of the composite to be 1.56 and that of aluminum to be 2.7, the thickness of the aluminum is

$$t_2 = \frac{1.56}{2.7} (0.386 \text{ mm}) = 0.223 \text{ mm} \quad (7.5)$$

Using the following equation (7.6) for the buckling strength of cylinders:

$$P_{cl} = C \frac{2\pi E t^2}{\sqrt{3(1-\nu^2)}} \quad (7.6)$$

where $C = 0.3$ gives

$$P_{cl} = 0.3 \frac{2\pi(70 \text{ GPa})(0.223 \text{ mm})^2}{\sqrt{3(1-0.3^2)}} = 13,233 \text{ N} \quad (7.7)$$

C is an empirical coefficient that accounts for the effect of imperfections. Experiments have shown that C varies between 0.22 and 0.48. Using these values, the critical load is in the range from 2,911 to 6,351 N. The A-cylinder has a buckling load of 12,600 N. This is between 4.32 and 1.98 times the load for the aluminum cylinder.

7.2.1.2 Composite cylinders made by filament winding

Almeida et al. [2] reported that results from filament wound tubes made of carbon/epoxy were tested under axial compressive loads. Tubes wound at $\pm 55^\circ$ have an average thickness of 0.56 mm and an outer radius of 68.56 mm. The maximum buckling load of this tube is 18,200 N.

Even though the A-cylinder has an outer radius of 71.6 mm, eq. (7.6) does not account for the effect of the radius; the critical load is proportional to the thickness

squared. Assuming that this relation holds, the equivalent buckling load for an A-cylinder with a thickness of 0.56 mm would be

$$P_{\text{equ}} = (12,600 \text{ N}) \left(\frac{0.56}{0.386} \right)^2 = 26,520 \text{ N} \quad (7.8)$$

Equation (7.8) shows that the buckling strength of the A-cylinder is 1.46 times (26,520/18,200) larger than that of a filament wound cylinder at $\pm 55^\circ$ with equivalent thickness. There are two reasons for this:

- Even though the filament wound cylinder has only two layers (55° and -55°), the layers are not compacted as much as prepregs. As such, the thickness of the filament wound layer is more than that of the prepreg layers. In addition, there are overlapping regions that increase the average thickness.
- The fiber angle at $\pm 55^\circ$ is not efficient in resisting the axial load. The most efficient angle would be 0° . The strength component along the axial direction is only a fraction of the strength along the fiber direction ($\sin^2 55^\circ = 0.67$). For an A-cylinder with a lay-up of $[0_2/90]$, the portion to resist the stress along the axial direction is $(2/3 = 0.67)$. If one were to use a laminate with a lay-up sequence of $[0_3/90]$, the efficiency of this laminate in resisting axial compression would be $3/4$.

The above shows that A-cylinders made by 4DPC have strong potential as axial buckling-resisting structural elements.

7.3 A-cones with circular cross section

7.3.1 Introduction

Cones are tapered cylinders. The section at one end is smaller than the section at the other end (Figure 7.8). Cones can be found in many applications such as the nose cone of an airplane, the nose cone of rockets, and the tail boom of helicopters. Cones serve the purpose of lightweighting or for the control of flow velocity and flow pressure.

Cones can be made of metals, paper, plastics, or long, continuous fiber composite materials. When metal (e.g., aluminum) is used, a sheet of metal is wrapped around a conical mandrel to form the cone. A special fixture is necessary (Figure 7.9). The two edges of the sheet metal are then welded together.

The procedure to make long fiber composite cones also requires a mold of conical shape. The mold can be made of either wood, metal, or plaster. The mold usually consists of two halves to facilitate the deposition of the composites. Composite fabrics are laid in two halves of the mold. The two halves are butt-joined to make the final cone. An overlap piece of fabric is applied over the butt joint to hold the two halves together.

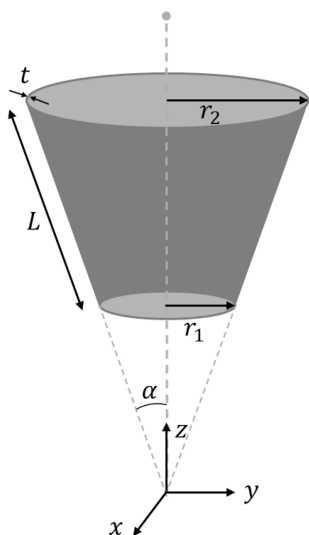


Figure 7.8: Cone configuration.



Figure 7.9: Fixture used to roll a metal sheet around a conical mandrel.

7.3.2 Manufacturing procedure using 4DPC

The following presents the manufacturing procedure for conical composite shells made by 4DPC. A more complete presentation, including the mechanical performance of the composite cones made by 4DPC, can be found in reference [3].

The procedure to make cones using 4DPC does not need a mold of conical shape. It consists of depositing layers of carbon/epoxy prepregs onto a flat mandrel. The composite strips are placed in curvilinear and radial directions, as shown in Figure 7.10. In Figure 7.10a, the first strip of composite is laid along the radial direction. Each strip of composite is made of prepreg of carbon/epoxy (or any other type of thermoset composite prepreg such as glass/epoxy or Kevlar/epoxy). The width of each strip can be

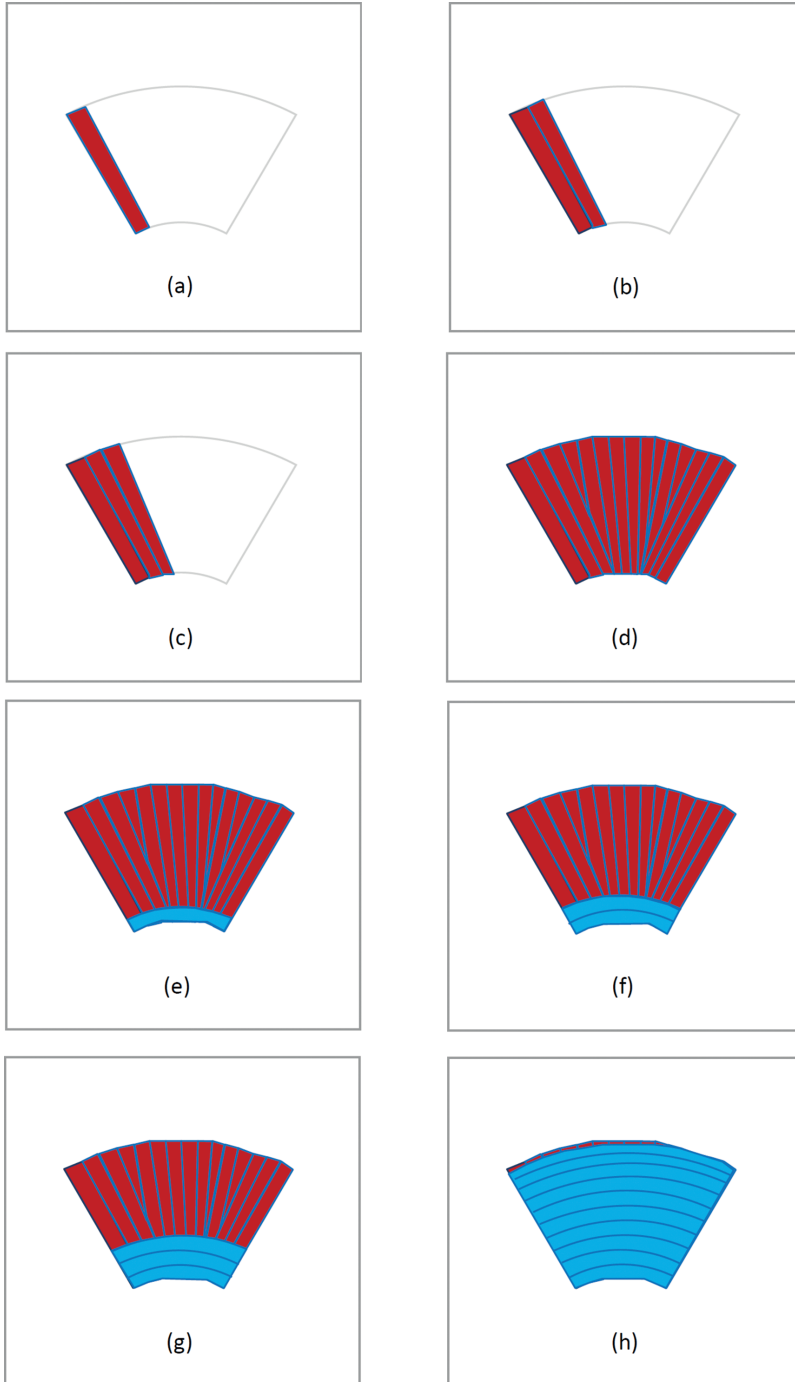


Figure 7.10: Deposition of radial strips (a, b, c, d) and circumferential strips (e, f, g, h) to make a flat stack.

1/8, 1/4, 1/2 in, or more, depending on the characteristics of the automated fiber placement machine. In Figure 7.10b, two strips along the radial direction have been deposited. The process continues until the whole surface of the curvilinear trapezoid is covered, as shown in Figure 7.10d. Figure 7.10e shows that the first strip along the circumferential direction has been deposited on top of the first layer. The process continues until the whole surface of the curvilinear trapezoid is covered with strips along the circumferential layer, as shown in Figure 7.10h. It can be seen that a laminate made of two layers has been made. The layers have fiber directions that are orthogonal to each other. This is similar to a [0/90] laminate for the case of straight fibers, except that one set of fibers is curved, and the other set of fibers has variable fiber orientation in this case. It should be noted that due to the tapered configuration of the blank, and the constant width of the tape, laps and gaps may occur during the filling up of the surface of the blank.

Figure 7.11 shows the case where a partial circumferential layer is laid on top of the radial layer. Figure 7.12 shows the stack of composite prepregs after deposition. Figure 7.13 shows a schematic of the flat laminate, which occupies the region *ABCD*.

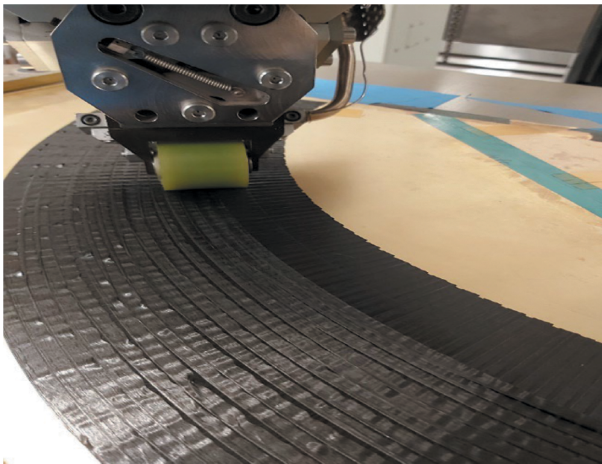


Figure 7.11: Circumferential strips laid on top of radial strips.



Figure 7.12: Prepregs laid down in curvilinear coordinates.

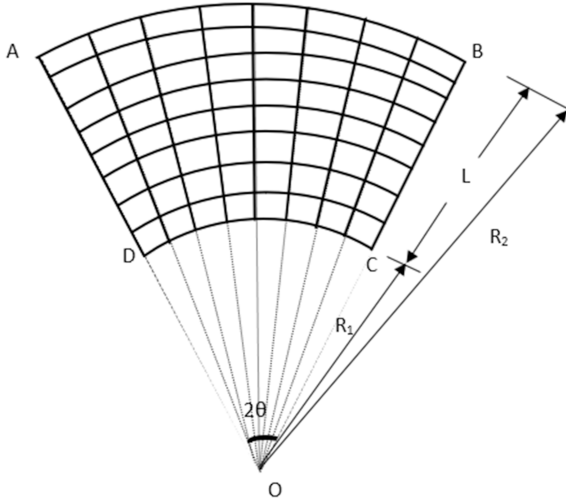


Figure 7.13: Schematics of the composite strips laid down using an automated fiber placement machine.

In Figure 7.13, R_1 represents the inner radius of the innermost strip and R_2 represents the outer radius of the outermost strip. The width of each strip is 0.25 in (6.35 mm). Note that there is a minimum limit for the radius R_1 . Below this limit, problems such as wrinkling may occur. The value of the lower limit depends on the tackiness of the tape, the temperature, the speed of deposition, and so on [4, 5].

If L represents the width of the laminate in Figure 7.13, then

$$R_2 = R_1 + L \quad (7.9)$$

The lengths CD and AB are given by

$$CD = 2R_1\theta \quad (7.10)$$

$$AB = 2R_2\theta \quad (7.11)$$

where 2θ is the included angle in radians.

The materials used to make the stack in Figure 7.12 are carbon/epoxy prepreps from Ten Cate. This was then vacuum-bagged and cured in an autoclave. The cure temperature was 135 °C for a duration of 90 min.

After curing and cooling to room temperature, the flat stack of composite prepreps becomes hard and curls itself into the shape of either a curved piece, as shown in Figure 7.14a (for short CD), or the shape of a cone when the length CD is long enough (Figure 7.14b).

It can be seen from the above description that 4DPC is a very simple process for the manufacturing of composite cones, as compared to current methods such as wrapping or filament winding. Not only is the process simple and does not require a mold

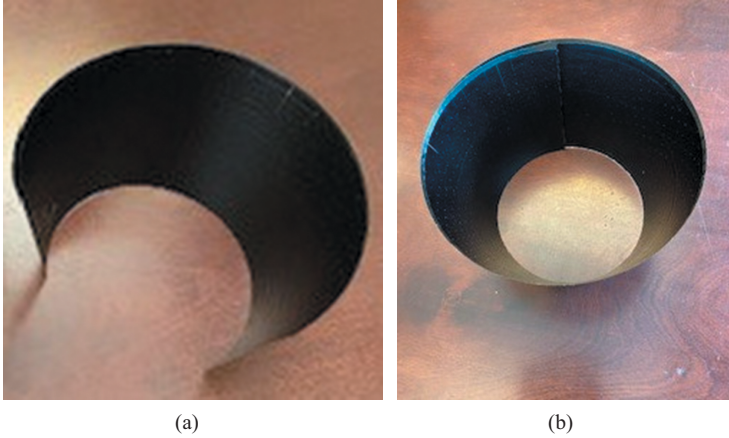


Figure 7.14: Deformed shape after curing and cooling: (a) curved piece and (b) cone configuration.

of conical shape, but these composite cones also have excellent resistance against buckling, as will be presented later.

7.3.3 Lay-up sequences

The procedure above describes the case for a laminate made of two layers: one layer along the radial direction and one layer along the circumferential direction. Let R denote the radial direction and C denote the circumferential direction. For the laminate in the above section, the lay-up is denoted as $[R_1/C_1]$. It indicates that the radial layer is placed before the circumferential layer. Due to the tendency for the tows to wrinkle when the steering radius is small, it is better to lay down the radial layer before the circumferential layer. More layers can be placed to increase the thickness of the laminate. As such, a general lay-up sequence can be written as $[R_m/C_n]$, where m and n represent the numbers of layers along the radial and circumferential directions, respectively.

7.3.4 Relationship between dimensions of the flat stack and those of the cone shape

It is of interest to determine the length CD (Figure 7.13) of the flat stack in terms of those of a full cone (Figure 7.8), for the case where a full cone is formed. In Figure 7.8, r_1 represents the radius of the lower circle, and r_2 represents the radius of the upper circle of a full cone, and L is the length of the meridian. The degree of taper of the cone is represented by α . The height h of the cone is given as $h = L \cos \alpha$. Correlating between Figures 7.13 and 7.8, one has

$$CD = 2\pi r_1 \quad (7.12)$$

From eqs. (7.10) and (7.12):

$$\theta = \frac{\pi r_1}{R_1} \quad (7.13)$$

From Figure 7.8, the degree of taper of the cone is given as follows:

$$\sin \alpha = \frac{r_1}{R_1}$$

giving

$$\theta = \pi \sin \alpha \quad (7.14)$$

If a complete cone with a half cone angle α is desired, then the covered angle 2θ for the flat stack needs to be obtained from eq. (7.14). The relations between the radii are

$$R_1 = \frac{r_1}{\sin \alpha} \quad R_2 = \frac{r_2}{\sin \alpha} \quad (7.15)$$

In the normal procedure of making a cone out of paper, plastic, or metal, a blank in the shape of the curved trapezoid is first cut out. It is then wrapped around a conical mandrel to make the cone. To make a cone with r_1 , r_2 , and α as given, the parameters θ , R_1 , and R_2 for the curved trapezoid are obtained from eqs. (7.14) and (7.15). However, in the case where manufacturing is done using 4DPC, the wrapping action is performed by the material itself, rather than by some external mechanism, as in the conventional method. As such, two trapezoids with the same R_1 , R_2 , and θ may curl up into different shapes if the lay-up sequences are different. The shape can be just a partial curved plate, a complete cone, or more than a complete cone. The shape depends on the lay-up sequence, the material properties, and the aspect ratio of the dimensions of the trapezoid itself.

As such, eqs. (7.12)–(7.15) may not hold for the case of 4DPC. For the case of A-cylinders, these radii may be approximated using the laminate theory. For conical shells, this may not be possible. Finite element techniques need to be used, as will be presented later.

7.3.5 Mechanical testing for compression properties

It is of interest to see whether the A-cones can withstand sufficient mechanical loads (compressive load) for practical engineering applications. Table 7.1 shows the specifics of the four cones tested. Note that r'_1 , r'_2 , and α' are the radii and cone angle obtained from manual bending of the shell into the cone to make overlap for adhesive bonding. The percent overlap region (also shown) was made to facilitate the bonding. Also shown in the table are specifics of aluminum cones. These are used to make a compar-

Table 7.1: Specifications of the four cones tested.

Cone no.	1	2	3	4	Aluminum (thin)	Aluminum (thick)
Lay-up	$[R_1/C_1]$	$[R_1/C_1]$	$[R_2/C_1]$	$[R_2/C_1]$		
R_1 (cm)	29.2	29.2	29.2	29.2		
R_2 (cm)	41.9	41.9	41.9	41.9		
2θ (degree)	85	85	80	80		
r'_1 (cm)	6.5	6.4	6.0	6.0		
r'_2 (cm)	8.6	8.4	7.6	7.7		
Visible height H (cm)	9.3	9.2	9.5	9.6		
Thickness t (mm)	0.23	0.24	0.32	0.32	0.23	0.32
α' (degrees)	12.8	12.3	9.5	10	12.8	9.5
Mass m (g)	33	33	43	44		
Average overlap (%) (cm)	8% (3.8)	8% (3.8)	3% (1.3)	3% (1.3)		
Buckling load (N)	4,582	6,092	15,089	16,575	4,942	9,674

ison between the experimental buckling loads of the composite A-cones and the theoretical loads of the isotropic aluminum cones.

The edges of the cones were bonded using epoxy adhesive. Figure 7.15 shows the bonding of the overlapping region in the cone.

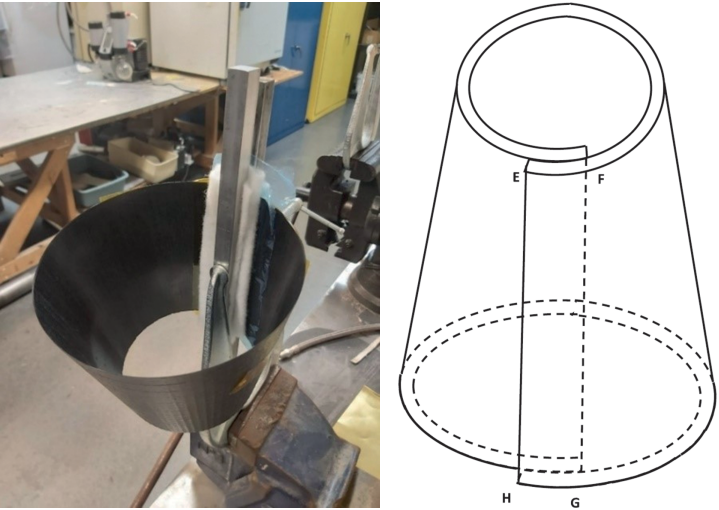


Figure 7.15: Bonding of the overlap region using epoxy adhesive.

Since the cone has very small thickness, direct contact between the loading plates and the cone edges may crush the cone at very low loads. These edges are protected using two loading plates made of aluminum. Each loading plate has a groove, as shown in Figure 7.16.

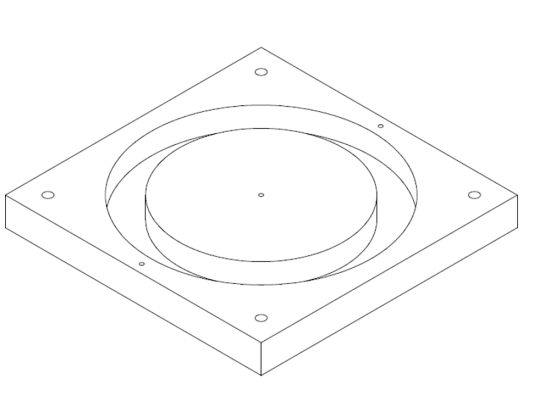


Figure 7.16: Grooves in the loading plate.

The cones are pressed against the edges of the loading plate within the grooves. Low melting point alloy material was poured into the groove to keep the cone in position. Strain gauges (T gauges) were bonded to cone 3 for strain measurements. In the T gauges, one strain element is along the direction of the meridian, while the other is along the circumferential direction.

The cone assembly was placed inside an MTS machine for compression load testing (Figure 7.17). Testing was done at a rate of 0.5 mm/min.

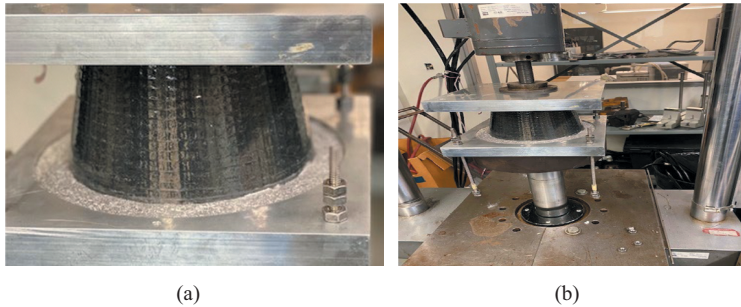


Figure 7.17: Cone in groove and assembly in the MTS machine.

7.3.5.1 Maximum buckling loads

Figures 7.18 and 7.19 show the load/displacement curves for cones 1 and 2. Figures 7.20 and 7.21 show the load/displacement curves for cones 3 and 4.

It can be seen that for cone 1, the maximum buckling load is 4,582 N for both the first loading and the second loading. The maximum load for cone 2 is 6,092 N for the first loading and 5,800 N for the second loading. Even though both cones 1 and 2 are made of the same lay-up sequence $[R_1/C_1]$, the difference in strength is due to the variation in the

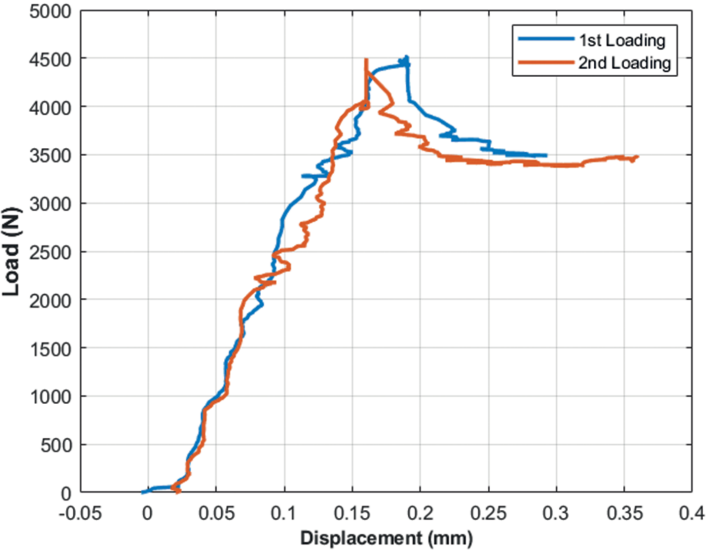


Figure 7.18: Load/displacement curves for cone 1 $[R_1/C_1]$.

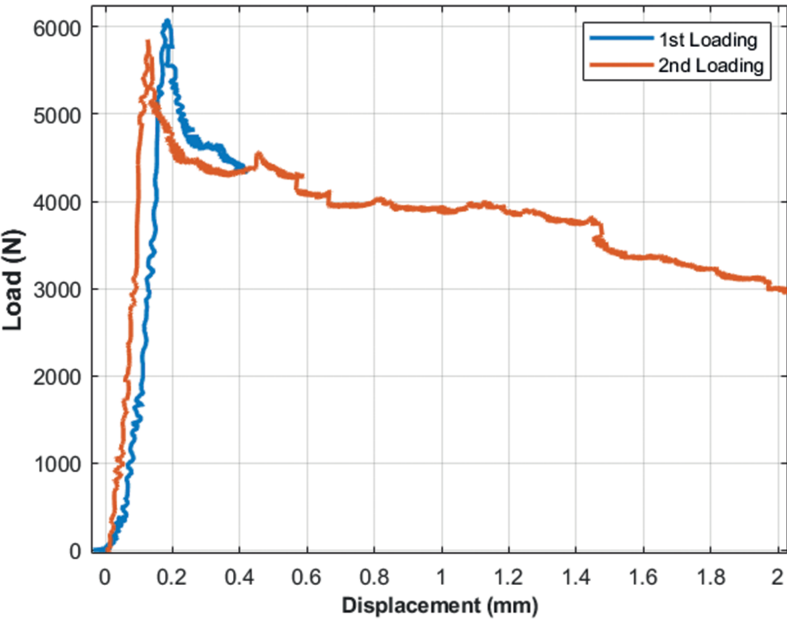


Figure 7.19: Load/displacement curves for cone 2 $[R_1/C_1]$.

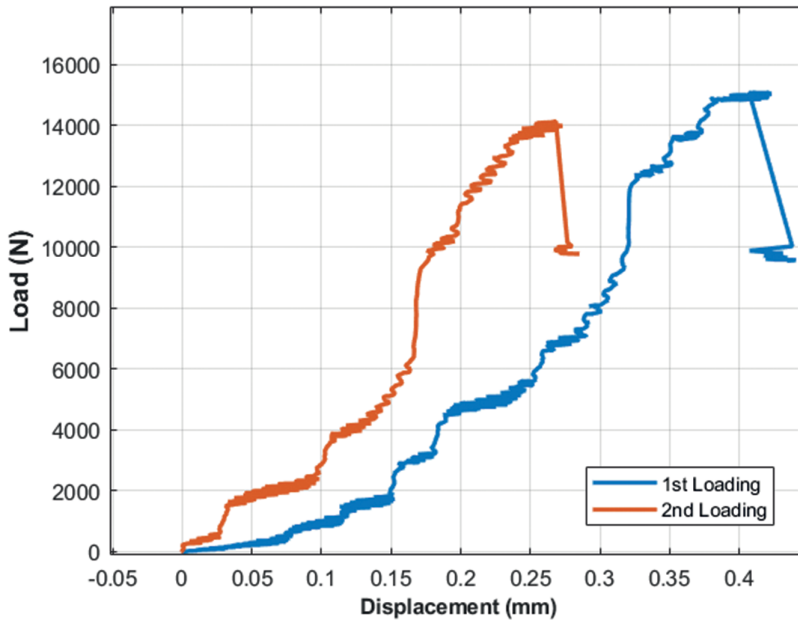


Figure 7.20: Load/displacement curves for cone 3 $[R_2/C_1]$.

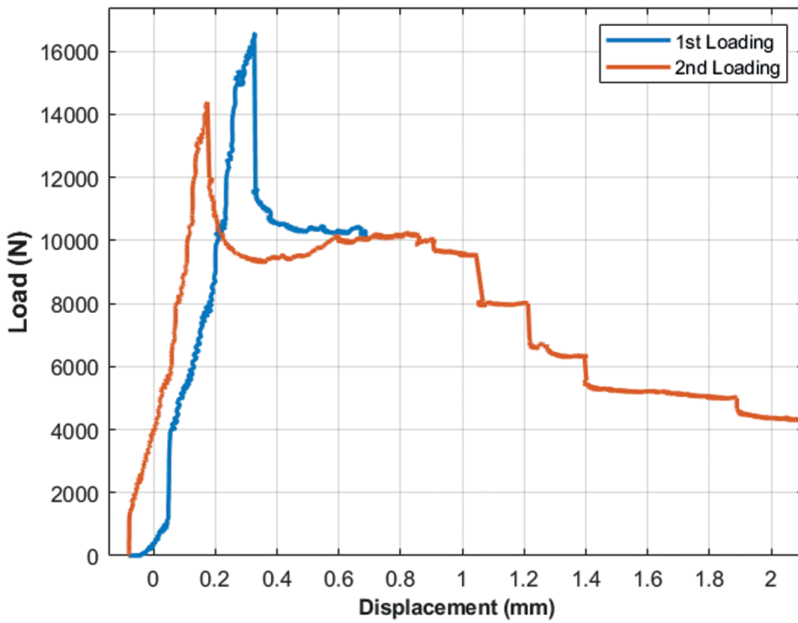


Figure 7.21: Load/displacement curves for cone 4 $[R_2/C_1]$.

manufacturing process. Cone 2 was improved from experience with cone 1. Cone 3 shows a buckling load of 15,089 N in the first loading. The second loading shows a lower buckling load and lower deformation due to damage done during the first loading. Cone 4 shows a maximum buckling load of 16,575 N from the first loading.

There is significant retention of resistance post-buckling. This aspect can be used for energy absorption capacity.

It would be of interest to compare the buckling loads from the four cones with cones made of aluminum having similar thickness. The equation for the buckling load of cones made of isotropic material is given as [6]

$$P_{cl} = C \frac{2 \pi E t^2}{\sqrt{3(1-\nu^2)}} \cos^2 \alpha \quad (7.16)$$

where E is the Young's modulus, t is cone thickness, ν is Poisson's ratio, and α is the half cone angle. C is a coefficient to take into consideration the effect of imperfections. Experiments on many cones show a C value of about 0.36. Aluminum has $E = 70$ GPa and $\nu = 0.3$. The buckling loads for aluminum are calculated using eq. (7.16) and are shown in Table 1.1.

7.3.6 Discussion

- The average buckling load for two cones made of two layers (R_1/C_1) is 5,337 N, while that for a cone made of aluminum with the same thickness is 4,942 N. The difference is 8%. Considering the specific gravity of aluminum to be 2.2 and that of the composite to be 1.8, the composite cone is 22% lighter. Taking into account the simple method of manufacturing the composite cone, composite cones have advantages.
- The average buckling load for two cones made of three layers (R_2/C_1) is 15,832 N, while that of a cone made of aluminum with the same thickness is 9,674 N. The difference is 63.6%. This is even more impressive than the case of the thinner cone.
- Increasing the thickness of the cone from two layers (R_1/C_1) to three layers (R_2/C_1) increases the buckling load from 5,337 to 15,832 N. This is an increase of 2.96 times. If one were to go with eq. (7.17), where the buckling load increases by the square of the thickness, the increase would be 2.25. The better-than-squared performance is due to the fact that the $[R_2/C_1]$ lay-up sequence has twice the amount of fibers along the meridian compared to the case of $[R_1/C_1]$.
- While fibers along the circumferential (C) direction serve to support the fibers along the radial (R) direction, the resistance against the compression load comes mainly from fibers along the radial direction.

- The reason for this excellent performance of the composite cone is due to the fact that all radial fibers contribute equally to the resistance against the compression load. If the cone were to be made using the conventional way of laying up straight fiber fabrics on a curvilinear trapezoid, as shown in Figure 7.22, then only a few fibers (those shown in region A) would be along the direction of the meridian of the cone, while the other fibers (e.g., region B) are at different orientations. As such, different fibers would have different contributions against the compression load, making the resistance ineffective.

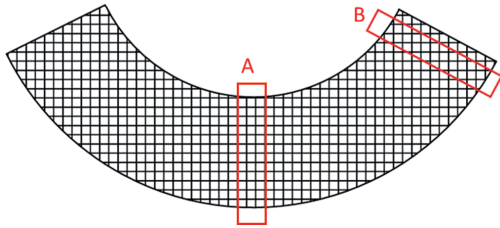


Figure 7.22: Straight fiber fabrics on a curvilinear trapezoid.

7.3.7 Strain results

Strain gauges were placed on the outer surface of cone 3. The strain results are shown in Figures 7.23 and 7.24. Each figure shows the strains from the first loading (at about 230 s) and the second loading (at about 470 s). For the first loading, in the case of circumferential strain (ϵ_θ), there is a peak of $-2,300 \mu\epsilon$, followed by a second peak of about $-1,600 \mu\epsilon$. In the case of axial strains (ϵ_x), there is a peak of $-4,200 \mu\epsilon$, followed by a positive strain of $1,700 \mu\epsilon$. These strain values show that initially the whole structure is under compression. After buckling, most of the resistance is along the circumferential direction. This can be seen from the significantly high values of the circumferential strain in the second loading, while the axial strain is practically 0 in the second loading. This can also be seen better by observing the buckling shape in Figures 7.25 and 7.26. Note that the strain readings may vary from location to location due to the local nature of the buckling mode shape.

7.3.7.1 Failure mode

Figures 7.25 and 7.26 show the failure modes.

The failure mode shows a large number of dimples. There are two rows of dimples along the vertical direction and seven columns of dimples along the circumferential direction. The rows and columns are not aligned; rather, they are alternating. The dimples show regions of compression and tension. Even though the failure seems to be extensive, there is remarkable recovery of the shape and resistance after unloading. This is evident from the strain values during the second loading.

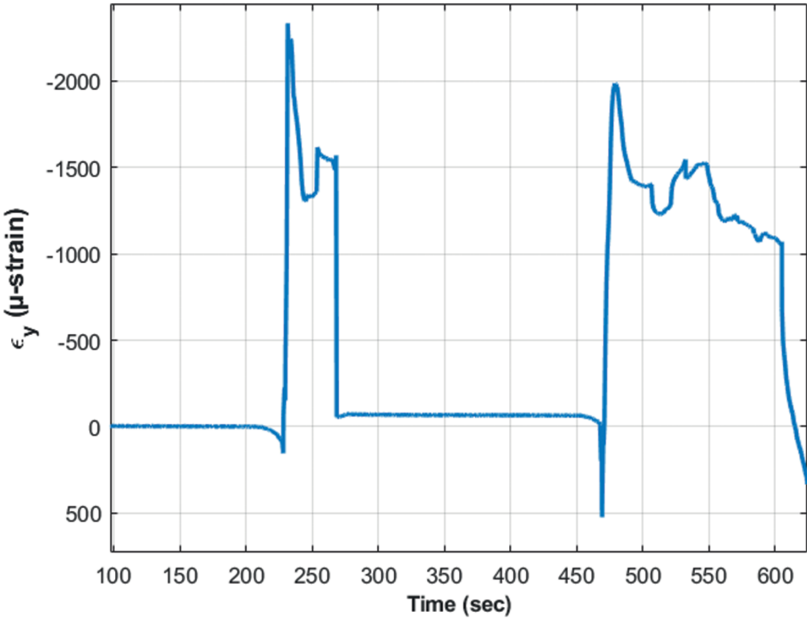


Figure 7.23: Circumferential strain results from cone 3.

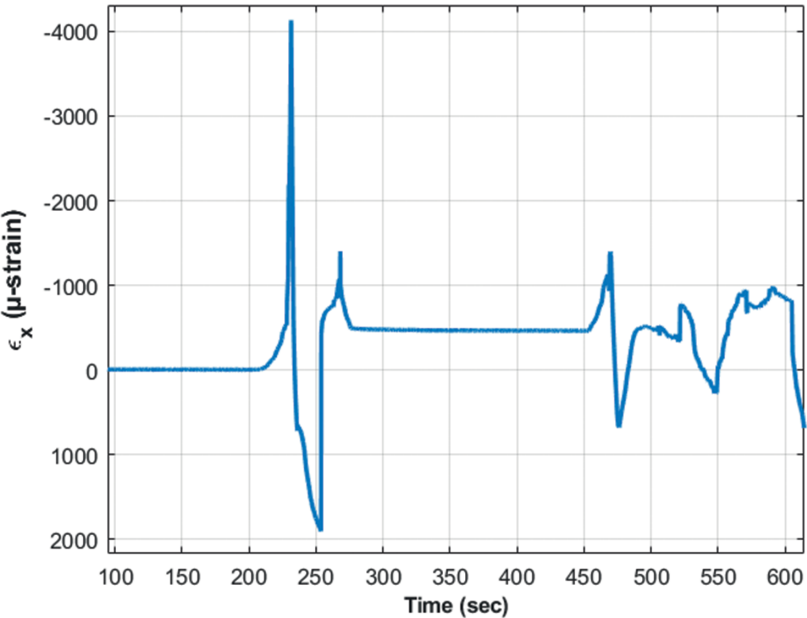


Figure 7.24: Axial strain results from cone 3.

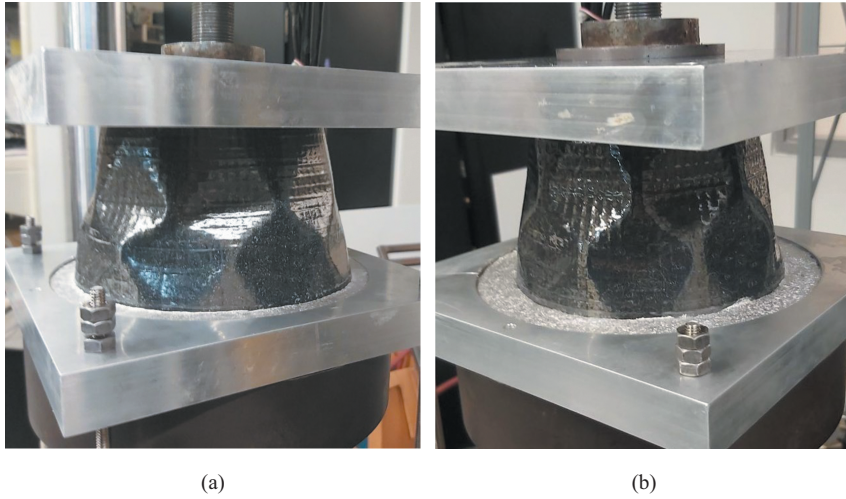


Figure 7.25: Failure mode right after buckling: (a) (R_1/C_1) and (b) (R_2/C_1) .

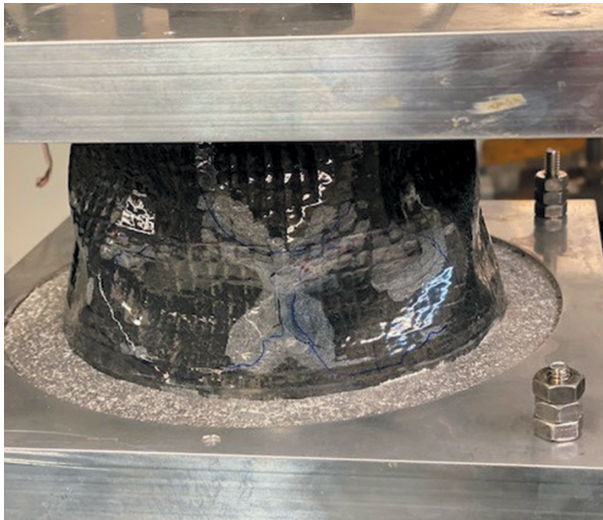


Figure 7.26: Sustained resistance after buckling failure.

7.3.8 Finite element method for the determination of the deformed shape

Since some of the fibers used in making the A-cone are curved, it is not possible to use laminate theory to determine the final deformed shape. Instead, the finite element method needs to be used. There are two ways of modeling the transformation

of the shape from flat to curved. The conventional way is to use elements of straight fibers. Most, if not all, finite element methods (as evident in package programs such as ANSYS and ABAQUS) for composites are for straight fibers. Figure 7.27 shows the configuration of a laminate consisting of a layer of radial fibers and a layer of circumferential, curved fibers.

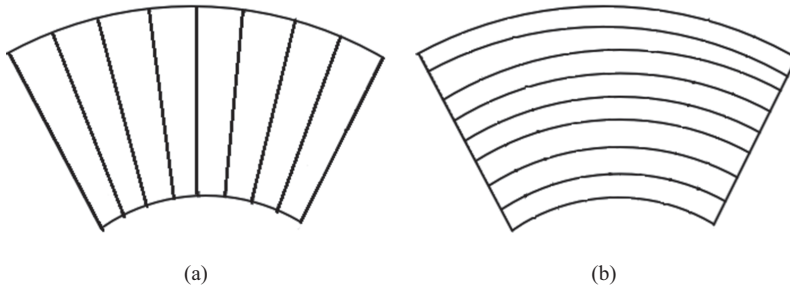


Figure 7.27: A laminate of two layers: (a) one layer with straight fibers along the radial direction and (b) the other layer with curved fibers in the circumferential direction.

For the case of the layer with curved fibers (Figure 7.28a), it may be represented by a mesh consisting of finite elements of straight fibers, as shown in Figure 7.28b, or by a mesh of finite elements of curved fibers, as shown in Figure 7.28c.

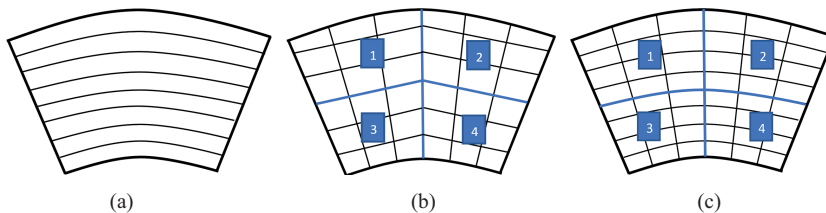


Figure 7.28: Finite element mesh (a) using straight fibers (b) or curved fibers (c).

The analysis using finite elements with curved fibers can be more efficient than the analysis using finite elements with straight fibers. This is shown in the presentation below.

The formulation of the finite element with curved fibers is achieved by considering the energy expression for each layer separately.

The weak form of the finite element formulation for this problem is derived, considering temperature-dependent material properties of composites. The temperature dependency is incorporated into the material properties, including the elastic moduli, shear moduli, and Poisson's ratios.

The lay-up for the structure is as shown in Figure 7.27. Note that here the lay-up consists of two parts, and each part has one layer. However, the procedure can be applied to

situations where each part has more than one layer with fibers along the same direction. The first layer has fibers along the radial direction, and the second layer has fibers along the circumferential direction. The combination of the two parts makes the laminate. The laminate is divided into a number of elements as shown in Figure 7.29.

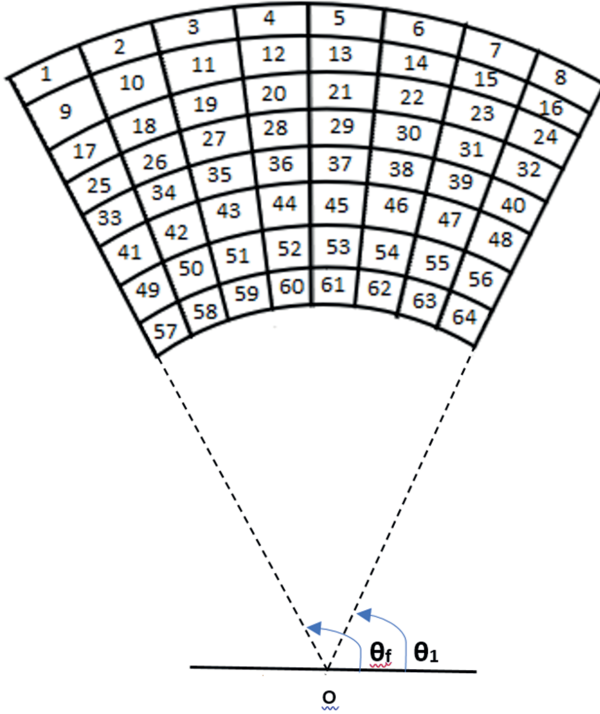


Figure 7.29: Element numbers.

Each element has four boundaries. Two of them are along the radial direction, one radial line at coordinate θ_i , and the other at coordinate θ_{i+1} . The other two are along the circumferential direction. One is at coordinate r_i and the other at coordinate r_{i+1} . For an element in the assembly, the range of θ varies from θ_i to θ_{i+1} . This range is equal to $(\theta_f - \theta_1)/m$ where m is the number of elements along the circumferential direction (Figure 7.30).

For assembling the stiffness matrices of the elements to make the global stiffness matrix, it is necessary that the element stiffness matrix be expressed using the same coordinate system. In this case, the coordinate system x, y, z with origin at O is chosen. Along this line, the element nodal coordinates need to be along the x, y , and z coordinate system, as shown in Figure 7.31. The displacement components u_i, v_i, w_i at each corner of the element are essentially the same u_{xi}, u_{yi}, u_{zi} , representing the displacements along three orthogonal directions specific to the element's local context. The

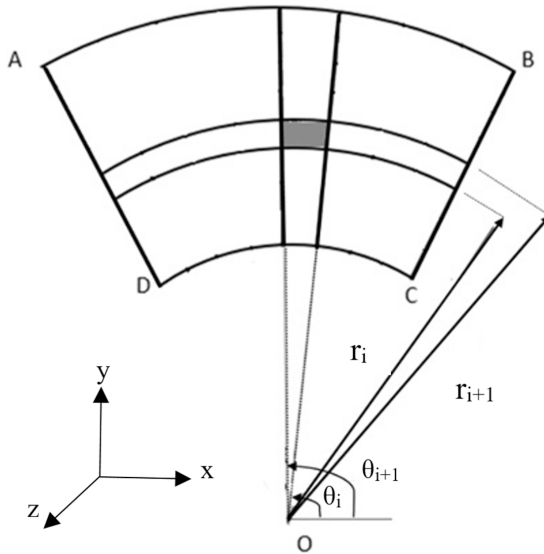


Figure 7.30: Location of an element.

geometrical and physical properties of the composite plate, designated as $ABCD$, determine its boundary and loading conditions, which influence the stress–strain state of the finite element $A'B'C'D'$. The conversion of local ($A'B'C'D'$) and global ($ABCD$) coordinate systems allows the integration of elemental responses with the composite plate's global behavior. This methodology allows for precise modeling of the plate's mechanical reaction under external loads, taking into account the complicated interplay between layers with different fiber orientations.

Note that four nodes are shown here, even though the element may have more nodes, if necessary.

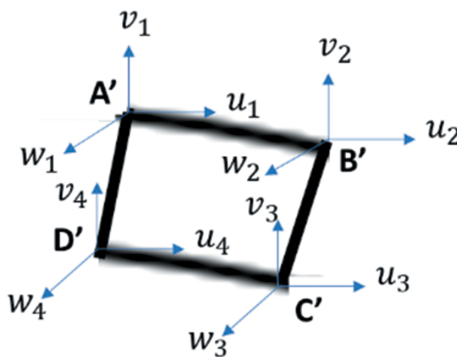


Figure 7.31: Nodal values for each element.

7.3.8.1 Strain energy expressions in terms of mechanical strains in material coordinates (r, θ, z) for plies 1 and 2 in each element: U_1 and U_2

The strain energy for ply 1 (fibers along the radial direction) is

$$U_1 = \int_1 \varepsilon_1^{1T} C_1 \varepsilon_1^1 dV \quad (7.18)$$

where ε_1^1 is the mechanical strain matrix in the material coordinate system, expressed in full as

$$\varepsilon_1^1 = (\varepsilon_r \quad \varepsilon_\theta \quad \varepsilon_z \quad \varepsilon_{rz} \quad \varepsilon_{\theta z} \quad \varepsilon_{r\theta})^T \quad (7.19)$$

Note that the material coordinate system for ply 1 is the same as the polar coordinates r, θ, z .

In ε_1^1 , the subscript refers to the ply number (1), and the superscript refers to the level of the transformation.

The stiffness matrix C or $[C]$ is expressed in full as follows:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (7.20)$$

Note that in eq. (7.11), $dV = r \, d\theta \, dz$. For composite materials, especially those used in laminates with fibers oriented in specific directions like radial and circumferential, the material stiffness matrices (C_1 and C_2) reflect the anisotropic nature of each ply. These matrices are defined in the material coordinate system, where the principal material directions are aligned with the fibers and perpendicular to them.

The strain energy for ply 2 is written as follows:

$$U_2 = \int_2 \varepsilon_2^{1T} C_2 \varepsilon_2^1 dV \quad (7.21)$$

Note in eq. (7.21), the stiffness matrix is the same as in eq. (7.18). The reason is that the equation is written in the material coordinates of ply 2.

The strains in ply 2 are expressed in cylindrical coordinates as follows:

$$\varepsilon_2^1 = (\varepsilon_\theta \quad \varepsilon_r \quad \varepsilon_z \quad \varepsilon_{\theta z} \quad \varepsilon_{rz} \quad \varepsilon_{r\theta})^T \quad (7.22)$$

7.3.8.2 Mechanical strains in terms of total strains and thermal strains in material coordinates

The strains in the previous section are mechanical strains. They need to be expressed in terms of total strains and thermal strains as follows:

$$(\epsilon_1^1)_{\text{mechanical}} = (\epsilon_1^1)_{\text{total}} - (\epsilon_1^1)_{\text{thermal}} \quad (7.23)$$

$$(\epsilon_2^1)_{\text{mechanical}} = (\epsilon_2^1)_{\text{total}} - (\epsilon_2^1)_{\text{thermal}} \quad (7.24)$$

where

$$(\epsilon_1^1)_{\text{thermal}} = (\Delta T) \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_2 & 0 & 0 & 0 \end{pmatrix}^T \quad (7.25)$$

$$(\epsilon_2^1)_{\text{thermal}} = (\Delta T) \begin{pmatrix} \alpha_2 & \alpha_1 & \alpha_2 & 0 & 0 & 0 \end{pmatrix}^T \quad (7.26)$$

α_1 , α_2 and α_3 are thermal expansion coefficients in the main directions.

7.3.8.3 Modify the expression for U_1 and U_2 in terms of total strains and thermal coefficients of thermal expansion

Substituting eqs. (7.23) and (7.24) into eqs. (7.18) and (7.21) gives

$$U_1 = \int_1 \left[(\epsilon_1^1)_{\text{total}} - (\epsilon_1^1)_{\text{thermal}} \right]^T C_1 \left[(\epsilon_1^1)_{\text{total}} - (\epsilon_1^1)_{\text{thermal}} \right] dV \quad (7.27)$$

$$U_2 = \int_2 \left[(\epsilon_2^1)_{\text{total}} - (\epsilon_2^1)_{\text{thermal}} \right]^T C_2 \left[(\epsilon_2^1)_{\text{total}} - (\epsilon_2^1)_{\text{thermal}} \right] dV \quad (7.28)$$

7.3.8.4 Transformation of total strains and thermal coefficients of thermal expansion from material coordinates (r, θ, z) to local coordinates (x, y, z)

At any point within the element (either for part 1 or part 2), there is an angle β between the x -axis and the fiber direction, as shown in Figure 7.31. For part 1, β is the same as θ . For part 2, β is equal to $(\theta - \pi/2)$. It can be seen that the range of β varies between the radial lines in Figure 7.31. The transformation of total strains and thermal coefficients of thermal expansion from material coordinates (r, θ, z) to local coordinates (x, y, z) is critical for the integration of material behaviors across different plies with varying fiber orientations.

7.3.8.4.1 Description of the transformation

Each ply in the laminate has fibers oriented differently, with the first layer aligned radially and the second circumferentially. The transformation matrices, $\bar{T}_1^1$ for the first layer and $\bar{T}_2^1$ for the second layer, based on the angle β in Figure 7.32, are utilized

to reorient the strain and thermal expansion vectors from their native polar coordinate systems to a shared Cartesian system suitable for unified analysis.

The limits of integration along the θ direction of an element would be between θ_i and θ_{i+1} , and the limits of integration along the r direction would be between r_i and r_{i+1} where

$$\Delta\theta = \theta_{i+1} - \theta_i = \frac{\theta_f - \theta_1}{m} \quad (7.29)$$

$$\Delta r = r_{i+1} - r_i = \frac{R_2 - R_1}{n} \quad (7.30)$$

where θ_1 , θ_f and R_1 , R_2 represent the boundaries of the trapezoid $ABCD$ along the circumferential direction and radial direction, respectively. Also, m and n are the number of elements along the circumferential and radial directions.

7.3.8.4.2 Mathematical formulation

In order to combine the two strain energies, it is necessary to convert the strains in cylindrical coordinates into the strains in common local coordinates x , y , z for plies 1 and 2 as follows:

$$(\epsilon_1^1)_{\text{total}} = \bar{T}_1^1 \epsilon_{x1} \quad (7.31)$$

$$(\epsilon_2^1)_{\text{total}} = \bar{T}_2^1 \epsilon_{x2} \quad (7.32)$$

$$(\epsilon_1^1)_{\text{thermal}} = \bar{T}_1^1 \alpha_{x1} \Delta T \quad (7.33)$$

$$(\epsilon_2^1)_{\text{thermal}} = \bar{T}_2^1 \alpha_{x2} \Delta T \quad (7.34)$$

where

$$\epsilon_{x1} = \begin{pmatrix} \epsilon_{x1} & \epsilon_{y1} & \epsilon_{z1} & \epsilon_{yz1} & \epsilon_{yz1} & \epsilon_{xy1} \end{pmatrix}^T \quad (7.35)$$

$$\epsilon_{x2} = \begin{pmatrix} \epsilon_{x2} & \epsilon_{y2} & \epsilon_{z2} & \epsilon_{yz2} & \epsilon_{yz2} & \epsilon_{xy2} \end{pmatrix}^T$$

$$\alpha_{x1} = \begin{pmatrix} \alpha_{x1} & \alpha_{y1} & \alpha_{z1} & \alpha_{yz1} & \alpha_{xz1} & \alpha_{xy1} \end{pmatrix}^T \quad (7.36)$$

$$\alpha_{x2} = \begin{pmatrix} \alpha_{x2} & \alpha_{y2} & \alpha_{z2} & \alpha_{yz2} & \alpha_{xz2} & \alpha_{xy2} \end{pmatrix}^T$$

α_{x1} and α_{x2} stand for the coefficient of thermal expansion in the local coordinate for layers 1 and 2.

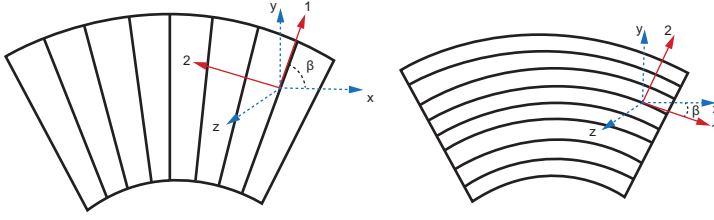


Figure 7.32: Relative orientation between material coordinates and local coordinates x, y, z .

7.3.8.5 Modify the expression for U_1 and U_2 to be in terms of the strain expressions in local coordinates (x, y, z) and express the strain energy for the element consisting of both plies

Substituting eqs. (7.31)–(7.34) into eqs. (7.27) and (7.28) gives the strain energy for an element, which consists of both plies 1 and 2 (U_1 and U_2) and the strain energy for the element, which consists of both plies 1 and 2 (U):

$$U_1 = \int \frac{1}{2} (\epsilon_{x1} - \alpha_{x1} \Delta T)^T [\bar{T}_1^1]^T [C_1] [\bar{T}_1^1] (\epsilon_{x1} - \alpha_{x1} \Delta T) dV = \int \frac{1}{2} (\epsilon_{x1} - \alpha_{x1} \Delta T)^T \bar{C}_1 (\epsilon_{x1} - \alpha_{x1} \Delta T) dV \quad (7.37)$$

$$U_2 = \int \frac{1}{2} (\epsilon_{x2} - \alpha_{x2} \Delta T)^T [\bar{T}_2^1]^T [C_2] [\bar{T}_2^1] (\epsilon_{x2} - \alpha_{x2} \Delta T) dV = \int \frac{1}{2} (\epsilon_{x2} - \alpha_{x2} \Delta T)^T \bar{C}_2 (\epsilon_{x2} - \alpha_{x2} \Delta T) dV \quad (7.38)$$

$$U = U_1 + U_2 = \frac{1}{2} \int \left((\epsilon_{x1} - \alpha_{x1} \Delta T)^T \bar{C}_1 (\epsilon_{x1} - \alpha_{x1} \Delta T) + (\epsilon_{x2} - \alpha_{x2} \Delta T)^T \bar{C}_2 (\epsilon_{x2} - \alpha_{x2} \Delta T) \right) dV \quad (7.39)$$

where $\bar{C}_i$ (ϵ^2) = $\bar{\sigma}_i$ and $\bar{C}_1 = [\bar{T}_1^1]^T [C_1] [\bar{T}_1^1]$ and $\bar{C}_2 = [\bar{T}_2^1]^T [C_2] [\bar{T}_2^1]$.

It can be seen that the $\bar{C}_i$ depends on the angle β , which in turn depends on the angle θ , and is shown in Figure 7.2. The angle θ varies from position to position within the element, and also the element strain is $\epsilon_x = \begin{pmatrix} \epsilon_x & \epsilon_y & \epsilon_z & \epsilon_{yz} & \epsilon_{xy} \end{pmatrix}^T$.

7.3.8.6 Define the global z -coordinate with the origin at the mid-thickness of the laminate

As shown in Figure 7.33, the z -coordinate for ply 1 is z_1 , and the limits of integration are from $-t_1/2$ to $t_1/2$. Also, the z -coordinate for ply 2 is z_2 , and the limits of integration are from $-t_2/2$ to $t_2/2$. In what follows, it will be assumed that $t_1 = t_2 = t$. For ply 1, the local z -coordinate is $z_1 = z - t/2$ and for ply 2, the local z -coordinate is $z_2 = z + t/2$.

This adjustment is crucial for simplifying the integration limits when calculating the strain energy and for correctly representing the strain distribution through the thickness of the laminate. The global z -coordinate is defined such that its origin is at

the mid-thickness of the laminate. Given that the laminate consists of both plies 1 and 2, and assuming each ply has an equal thickness t , the total thickness of the laminate is $2t$. Therefore, the global z -coordinate (z) ranges from $-t$ to t with the origin at 0, which corresponds to the mid-thickness of the laminate.

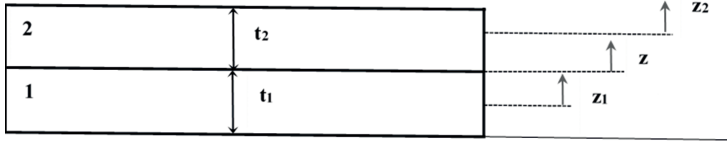


Figure 7.33: Coordinates along the z -direction.

7.3.8.7 Modification of strain energy expression integrating strain distribution across composite laminate

Given the new z -coordinate, the integration limits for calculating the strain energy will change. Let's denote U_1 as the strain energy for the first ply and U_2 as the strain energy for the second ply. The total strain energy U of the composite element can then be expressed as the sum of U_1 and U_2 . The modified expression for U , considering both plies and using the global z -coordinate system, is

$$\begin{aligned}
 U_1 &= \int_{-t_1/2}^{t_1/2} \frac{1}{2} (\epsilon_{x1} - \alpha_{x1} \Delta T)^T \bar{\mathbf{C}}_1 (\epsilon_{x1} - \alpha_{x1} \Delta T) dA \, dz_1 = \int_{-t_1}^0 (\epsilon_x - \alpha_{x1} \Delta T)^T \bar{\mathbf{C}}_1 (\epsilon_x - \alpha_{x1} \Delta T) dA \, dz \\
 U_2 &= \int_{-t_1/2}^{t_1/2} \frac{1}{2} (\epsilon_{x2} - \alpha_{x2} \Delta T)^T \bar{\mathbf{C}}_2 (\epsilon_{x2} - \alpha_{x2} \Delta T) dA \, dz_2 = \int_0^{t_2} (\epsilon_x - \alpha_{x2} \Delta T)^T \bar{\mathbf{C}}_2 (\epsilon_x - \alpha_{x2} \Delta T) dA \, dz
 \end{aligned} \tag{7.40}$$

This expression integrates the strain energy across the entire thickness of the laminate, from $-t_1$ to t_2 . ϵ_x is the total strain for the entire element in the local coordinates, and ϵ_{x1} and ϵ_{x2} are the total strains for plies 1 and 2, respectively. Then, the expression for total strain energy U , considering the thermal strains, would be

$$U = \frac{1}{2} \left(\int_{-t_1}^0 (\epsilon_x - \alpha_{x1} \Delta T)^T \bar{\mathbf{C}}_1 (\epsilon_x - \alpha_{x1} \Delta T) dA \, dz + \int_0^{t_2} (\epsilon_x - \alpha_{x2} \Delta T)^T \bar{\mathbf{C}}_2 (\epsilon_x - \alpha_{x2} \Delta T) dA \, dz \right) \tag{7.41}$$

Expressing the total strain components in local coordinates as functions of the nodal displacements, as follows:

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u^o}{\partial x} + \frac{1}{2} \left(\frac{\partial w^o}{\partial x} \right)^2 \\
\varepsilon_y &= \frac{\partial v^o}{\partial y} + \frac{1}{2} \left(\frac{\partial w^o}{\partial y} \right)^2 \\
\varepsilon_z &= \frac{\partial w^o}{\partial z} \\
\varepsilon_{xy} &= \left(\frac{\partial u^o}{\partial y} + \frac{\partial v^o}{\partial x} + \frac{\partial w^o}{\partial x} \frac{\partial w^o}{\partial y} \right) \\
\varepsilon_{xz} &= \left(\frac{\partial u^o}{\partial z} + \frac{\partial v^o}{\partial x} + \frac{\partial w^o}{\partial x} \frac{\partial w^o}{\partial z} \right) \\
\varepsilon_{yz} &= \left(\frac{\partial v^o}{\partial z} + \frac{\partial w^o}{\partial y} \frac{\partial w^o}{\partial z} \right) \\
k_{xx} &= -\frac{\partial^2 w^o}{\partial x^2} \\
k_{yy} &= -\frac{\partial^2 w^o}{\partial y^2} \\
k_{xy} &= -2\frac{\partial^2 w^o}{\partial x \partial y} \\
k_{xz} &= 0 \\
k_{yz} &= 0
\end{aligned} \tag{7.42}$$

u^o, v^o, w^o are the plane reference displacements and k_{ij} (i and j are local coordinates (x, y, z)) are curvatures.

In a three-dimensional element, the displacement field is approximated as follows:

$$\begin{aligned}
u^0(x, y, z) &= \sum_{i=1}^n N_i(x, y, z) \delta_{xi}^0 \\
v^0(x, y, z) &= \sum_{i=1}^n N_i(x, y, z) \delta_{yi}^0 \\
w^0(x, y, z) &= \sum_{i=1}^n N_i(x, y, z) \delta_{zi}^0
\end{aligned} \tag{7.43}$$

Here, δ_{xi}^0 , δ_{yi}^0 , and δ_{zi}^0 are the nodal displacements in the x , y , and z directions, respectively.

7.3.8.8 Using shape functions to express the displacements in terms of nodal displacements in local coordinates x, y, z

In a composite element that consists of multiple layers or plies, it would typically use the same nodal displacement vector $\delta_l = (\delta_x^0, \delta_y^0, \delta_z^0)$ for all layers when calculating the strain energy of the element. The reason for this is that the displacement vector δ_l represents the displacements at the nodes of the entire element, not individual plies. Since the nodes are shared by all plies, the displacements at those nodes are common to all layers.

The displacements $\mathbf{u} = (u, v, w)$ at each point of element $A'B'C'D'$ can be expressed in terms of the shape function $\mathbf{N}_e$ and the local nodal displacement δ_l as follows:

$$\mathbf{u} = \mathbf{N}_e \delta_l \quad (7.44)$$

It is essential to express strains in terms of displacements using the shape functions typically used in the finite element analysis. The strains in the laminate can be expressed as follows:

$$\varepsilon_x = \mathbf{B} \delta_l \quad (7.45)$$

where $\mathbf{B}$ is the strain–displacement matrix and is obtained by differentiating terms in $\mathbf{N}$. This relationship allows the direct calculation of strains from the known displacements at the nodes, facilitating the integration of mechanical behaviors into the overall structural analysis of the laminate.

From the strain–displacement relation, an expression between the strain and nodal displacement can be written as follows:

$$\varepsilon_x - \alpha_{x1} \Delta T = (\nabla \mathbf{N}_e) (\delta_l) - \alpha_{x1} \Delta T = \mathbf{B} \delta_l - \alpha_{x1} \Delta T \quad (7.46)$$

7.3.8.9 Express the strain energy U for the element in terms of local nodal displacements

The total strain energy, U , is then the sum of the two energies:

$$U = \frac{1}{2} \left(\int_{-t_1}^0 (\mathbf{B} \delta_l - \alpha_{x1} \Delta T)^T \bar{\mathbf{C}}_1 (\mathbf{B} \delta_l - \alpha_{x1} \Delta T) dA dz + \int_0^{t_2} (\mathbf{B} \delta_l - \alpha_{x2} \Delta T)^T \bar{\mathbf{C}}_2 (\mathbf{B} \delta_l - \alpha_{x2} \Delta T) dA dz \right) \quad (7.47)$$

This expression relates the strain energy of the element directly to the local nodal displacements through the shape functions.

Figure 7.34 illustrates the transformation of constitutive properties among global, local, and principal material coordinate systems for a circumferential ply.

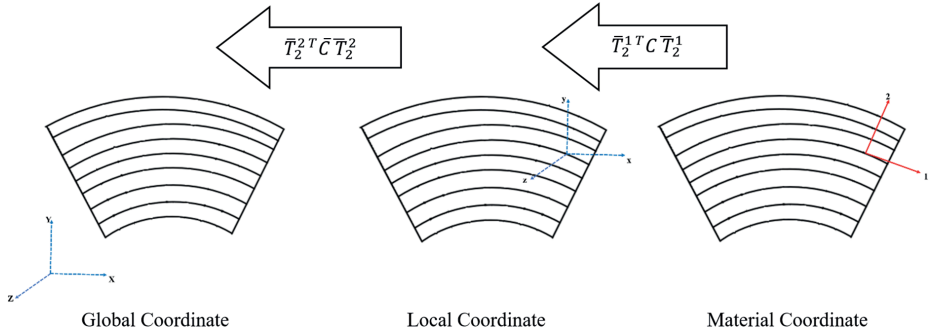


Figure 7.34: Transformation of constitutive properties between global, local, and principal material coordinate systems of circumferential ply.

7.3.8.10 Transform the local nodal displacements in x, y, z coordinates into the global nodal displacements in X, Y, Z coordinates

From Figure 7.31, it can be seen that the nodal displacement vectors have different orientations from one another. In order to facilitate the assembling process, these nodal displacements need to be transformed into the global nodal displacements δ_g in the X, Y, Z coordinates, as follows:

$$(\delta_l) = [\bar{T}^2] (\delta_g) \quad (7.48)$$

Substituting eq. (7.41) into eq. (7.40), an expression for the strain energy of an element in terms of global nodal displacements can be obtained. The assembly process can be performed to obtain the equilibrium equation for the structure.

The transformation matrix $[\bar{T}^2]$ can be expressed as follows:

$$[\bar{T}^2] = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7.49)$$

where θ is the angle between the local x -axis and the global X -axis, measured in a counterclockwise direction from the global X -axis to the local x -axis.

The strain energy of an element in global coordinates is

$$U_e = \frac{1}{2} \left(\int_{-t_1}^0 (\mathbf{B}\bar{\mathbf{T}}^2 \delta_g - \alpha_{x1}\Delta T)^T \bar{\mathbf{C}}_1 (\mathbf{B}\bar{\mathbf{T}}^2 \delta_g - \alpha_{x1}\Delta T) dA dz + \int_0^{t_2} (\mathbf{B}\bar{\mathbf{T}}^2 \delta_g - \alpha_{x2}\Delta T)^T \bar{\mathbf{C}}_2 (\mathbf{B}\bar{\mathbf{T}}^2 \delta_g - \alpha_{x2}\Delta T) dA dz \right) \quad (7.50)$$

With expanding eq. (7.50):

$$U_e = \frac{1}{2} \left(\left(\int_{-t_1}^0 \begin{pmatrix} (\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_1 \mathbf{B} [\bar{T}^2] \delta_g \\ -(\delta_g)(\alpha_{x1}\Delta T)^T \bar{\mathbf{C}}_1 \mathbf{B} [\bar{T}^2] \\ -(\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_1 (\alpha_{x1}\Delta T) \\ +(\alpha_{x1}\Delta T)^T \bar{\mathbf{C}}_1 (\alpha_{x1}\Delta T) \end{pmatrix} + \int_0^{t_2} \begin{pmatrix} (\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_2 \mathbf{B} [\bar{T}^2] (\delta_g) \\ -(\alpha_{x2}\Delta T)^T \bar{\mathbf{C}}_2 \mathbf{B} [\bar{T}^2] (\delta_g) \\ -(\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_2 (\alpha_{x2}\Delta T) \\ +(\alpha_{x2}\Delta T)^T \bar{\mathbf{C}}_2 (\alpha_{x2}\Delta T) \end{pmatrix} dA dz \right) \right) \quad (7.51)$$

$$U_e = \frac{1}{2} \left(\left(\left(\int_{-t_1}^0 \left((\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_1 \mathbf{B} [\bar{T}^2] (\delta_g) \right) + \int_0^{t_2} \left((\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_2 \mathbf{B} [\bar{T}^2] (\delta_g) \right) \right) dA dz \right) - \left(\left(\int_{-t_1}^0 \left((\delta_g)(\alpha_{x1}\Delta T)^T \bar{\mathbf{C}}_1 \mathbf{B} [\bar{T}^2] + (\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_1 (\alpha_{x1}\Delta T) \right) + \int_0^{t_2} \left((\delta_g)(\alpha_{x2}\Delta T)^T \bar{\mathbf{C}}_2 \mathbf{B} [\bar{T}^2] + (\delta_g)^T [\bar{T}^2]^T \mathbf{B}^T \bar{\mathbf{C}}_2 (\alpha_{x2}\Delta T) \right) \right) dA dz \right) + \left(\int_{-t_1}^0 \left((\alpha_{x1}\Delta T)^T \bar{\mathbf{C}}_1 (\alpha_{x1}\Delta T) \right) + \int_0^{t_2} \left((\alpha_{x2}\Delta T)^T \bar{\mathbf{C}}_2 (\alpha_{x2}\Delta T) \right) \right) dA dz \right) \right) \quad (7.52)$$

For each element e , the strain energy U_e can be calculated using the expression derived above. The strain energy in eq. (7.52) has three different components as follows:

- **Component 1:** Elastic strain energy. This component represents the basic elastic strain energy from mechanical deformations.
- **Combined component 2:** Interaction of mechanical and thermal strains. This expression assumes that both thermal strains and their interactions with mechanical strains.
- **Component 3:** Thermal expansion strain energy. This term calculates the strain energy due purely to thermal effects, without direct mechanical load interaction.

7.3.9 Comparison between straight fiber finite element method and curved fiber finite element method

The flat plate has a curvilinear geometry, representing a section of a hollow cylinder. The initial setup had $R_1 = 30.5$ mm, $R_2 = 50.5$ mm, $\theta_1 = 60^\circ$ and $\theta_f = 120^\circ$.

Figures 7.35 and 7.36 illustrate the curvature at the bottom and top lines of the plate with straight fiber orientations using different mesh sizes. In straight fiber composites, reducing the element size allows the curvature to more closely approximate the experimental outcomes observed in curvilinear composites, although it does not completely converge to these results. In straight fiber composite simulations, the computation time falls significantly as the number of elements decreases: from 44 min with 4,608 elements

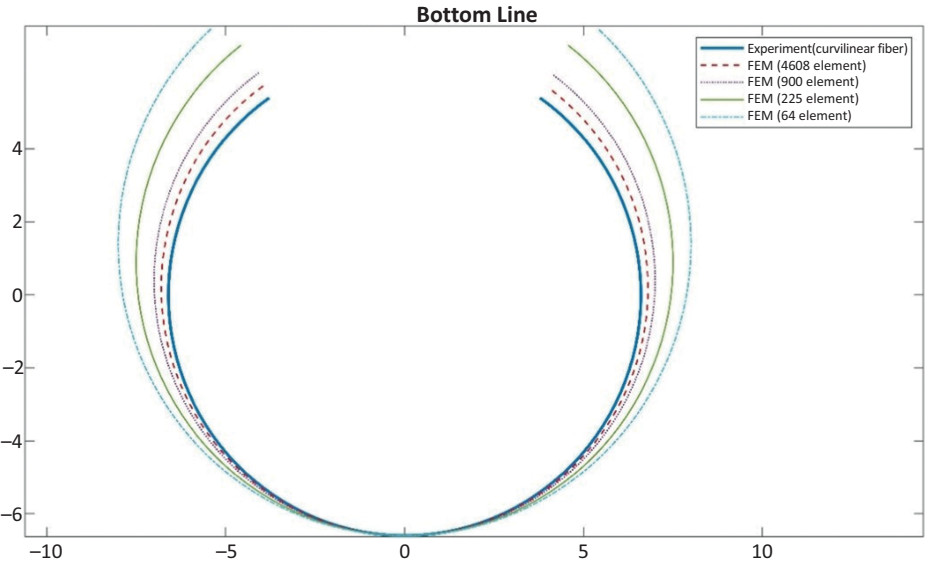


Figure 7.35: Comparison of finite element method with straight mesh and experimental results after cooling with straight fiber for the bottom line.

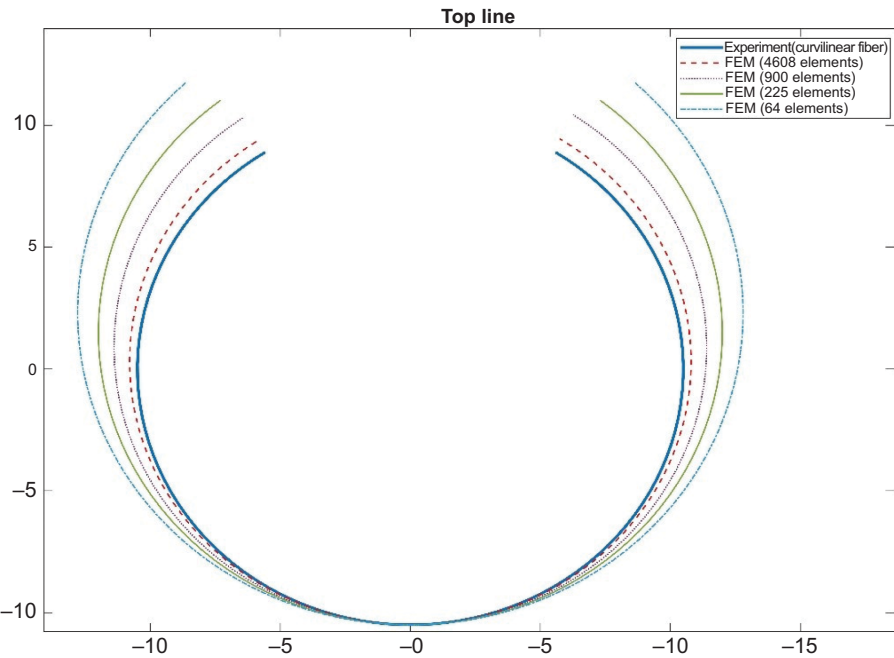


Figure 7.36: Comparison of finite element method with straight mesh and experimental results after cooling with straight fiber for the top line.

to 31 min with 900 elements, 18 min with 225 elements, and only 7 min with 64 elements. However, the accuracy also decreases rapidly.

Table 7.2 shows how the two finite element techniques differ in terms of the number of elements, mesh size, and overall computation time. Finite elements with straight fibers take 44 min and still cannot converge to experimental results, while finite elements with curved fibers take 19 min with good convergence to experimental results.

Table 7.2: Comparison of finite element analysis parameters for straight and curvilinear fiber models.

	Number of elements	Mesh size	Calculation time*
FE with straight fibers	4,608	96 $L \times 48 W$	44 min
FE with curved fibers	256	16 $L \times 16 W$	19 min

*Using Core i7 and 16 GB RAM.

7.4 Conclusion

It can be seen from the above description that 4DPC is a very simple process for the manufacturing of A-structures, either A-cylinders or A-cones, as compared to current methods. These structures also have good resistance against axial buckling.

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Part III: **Design space**

Chapter 8

Expansion of design space

The structures given in this chapter have not been created but are included to give ideas for future investigations.

Due to the fact that 4D printing of composites (4DPCs) relies on the design of the lay-up sequence, material properties, and the difference between the cure temperature and room temperature to obtain the desired shape, there is some concern about the extent of the design space. The design space can be expanded by methods that can affect the radius of curvature, by methods that can affect the mechanical properties, and by methods that can provide other types of structures. These methods are presented below.

8.1 Methods to affect the radius of curvature

8.1.1 Decreasing the radius of curvature

Methods to decrease the radius of curvature can be as follows:

- Using composite materials with a higher degree of anisotropy would mean that the ratios between E_1/E_2 and α_2/α_1 can be higher. This can be a challenge in finding new types of material. A soft matrix such as rubber can be used to provide a larger α_2/α_1 ratio. However, this can be at the expense of losing stiffness.
- Using thin plies, eqs. (1.88)–(1.92) show that the curvature is proportional to $1/h$, where h is the thickness of the individual layer. A smaller h value would result in larger curvature (or a smaller radius of curvature). One way to achieve this is by using composite prepregs with thin plies. There are composite prepregs with ply thicknesses as thin as 0.02 mm [1]. This is compared with a thickness of about 0.125 mm for conventional composite prepregs. For composite plies that are 6 times thinner than conventional plies, A-cylinder with a radius of curvature that is 6 times smaller may be obtained. Section 5.3 showed that a laminate with a lay-up sequence [0/90] would give a radius of curvature of 6.3 cm. Using thin plies of thickness of 0.02 mm would give a radius of 1.01 cm.
- Figure 8.1 shows the different types of lay-up arrangements. The top figure shows a lay-up sequence of [0/90]. In the normal way of lay-up (say using the hand lay-up method), layer 1 would be made up of all fibers along the x direction, while layer 2 (as shown in layer 2a) would be made up of all fibers along the y direction. However, if one were to use an automated fiber placement machine (hand lay-up can also be used, but the work is more laborious), one can lay the strips along the y direction as shown in layer 2b. In layer 2b, not all strips are laid down. Instead, only half of them are laid down. There is a space of the same width as that of a tow between two adjacent strips.

For a laminate with a lay-up sequence $[0/90]$ and the 90° layer is that of Figure 8.1 – layer 2b, the amount of material in layer 2 is only half as much as usual. One can assume that this is equivalent to a 90° layer with half the normal thickness. The lay-up sequence can be represented as $[0_2/90]$, where the thickness of each layer is now half the thickness of a normal layer. The radius of curvature of this new laminate would be half that of the normal $[0_2/90]$ as follows:

$$R_2 = \frac{1}{2} R_1 \quad (8.1)$$

where R_2 is the radius of curvature of laminate $[0_2/90]_{h/2}$ and R_1 is the radius of curvature of laminate $[0_2/90]_h$.

If the laminate is made of a full 0° layer in layer 1, and the second layer along the 90° direction is as shown in Figure 8.1 – layer 2c, then the thickness of the second layer is equivalent to $1/3$ the thickness of a normal layer. One then has

$$R_3 = \frac{1}{3} R_1 \quad (8.2)$$

where R_3 is the radius of curvature of the laminate $[0_3/90]_{h/3}$.

For the purpose of comparing the radius of curvature, Table 8.1 and Figure 8.3 show the values of the radius of curvature with variation in the n value for laminates with the stacking sequence $[0/90_n]$.

Table 8.1 shows that the radius of curvature of an A-cylinder made from a lay-up sequence of $[0/90_2]$ is 6.1 cm for a layer thickness of 0.125 mm. When composites of thin ply (ply thickness of 0.02 mm) are used, the radius of curvature would be 0.976 cm. If the arrangement using layers 2a and 2b in Figure 8.1 is used, the radius of curvature would be reduced to half (i.e., 0.488 cm). If the arrangement using layers 2a and 2c in Figure 8.1 is used, the radius would be reduced to one third (i.e., 0.325 cm). Figure 8.2 shows the relative dimensions of the different A-cylinders.

Since the structure is fairly thin, the A-cylinder can be bent to an even smaller radius if necessary.

8.1.2 Changing the radius of curvature by modifying the lay-up sequence

The radius of curvature of the final piece can be changed with the lay-up sequence. This is illustrated using the family of laminates $[0_m/90_n]$. Using eqs. (1.88), (1.90), (1.91), and (1.92) and the material properties shown in Table 1.1, the radii of curvature for laminates with different m and n are calculated. They are listed in Table 8.1 and Figure 8.3.

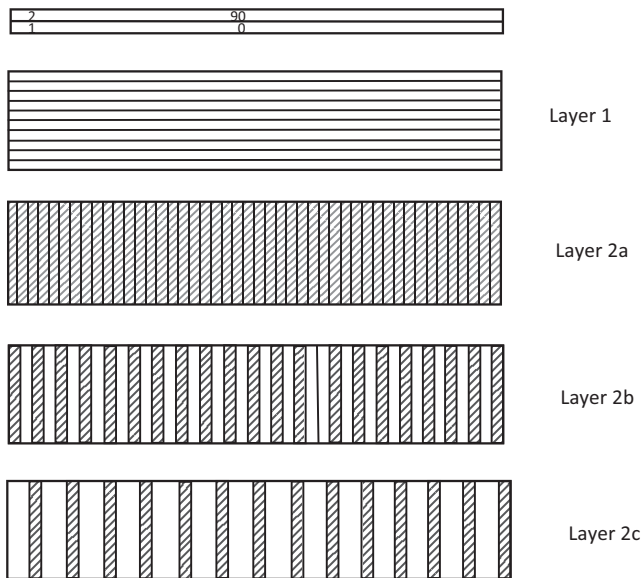


Figure 8.1: Different types of lay-up arrangements.

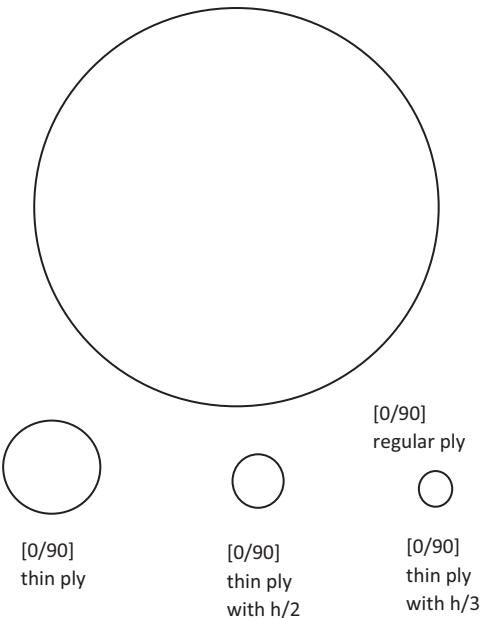


Figure 8.2: Relative diameters of the A-cylinders.

Table 8.1: Radius of curvature of $[0/90_n]$ laminates with different n values ($h = 0.125$ mm).

Lay-up	Thickness (mm)	Calculated radius of curvature (cm) (from laminate theory)
$[0/90]$	0.250	6.3
$[0/90_2]$	0.375	6.1
$[0/90_3]$	0.500	7.6
$[0/90_4]$	0.625	9.5
$[0/90_5]$	0.750	11.5
$[0/90_6]$	0.875	13.7
$[0/90_7]$	1.000	15.9
$[0/90_8]$	1.125	18.4
$[0/90_9]$	1.250	20.8
$[0/90_{10}]$	1.375	23.4
$[0_2/90]$	0.375	20.1
$[0_2/90_2]$	0.500	12.5
$[0_2/90_3]$	0.625	11.6
$[0_2/90_4]$	0.750	12.3
$[0_2/90_5]$	0.875	13.6
$[0_2/90_6]$	1.000	15.2
$[0_2/90_7]$	1.125	17.0
$[0_2/90_8]$	1.250	18.9
$[0_2/90_9]$	1.375	21.0
$[0_2/90_{10}]$	1.500	23.1
$[0_8/90_{12}]$	2.500	46.4
$[0_{16}/90_{24}]$	5.000	93
$[0_{24}/90_{36}]$	7.500	139

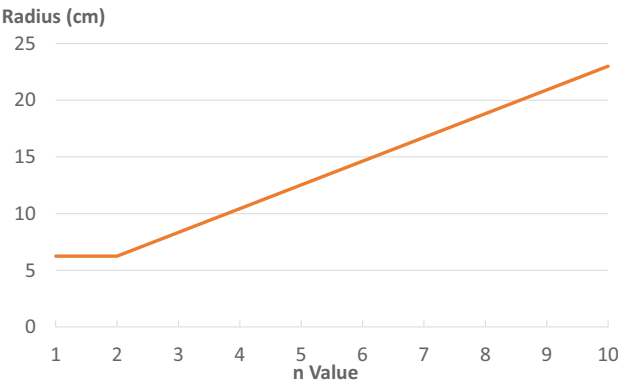
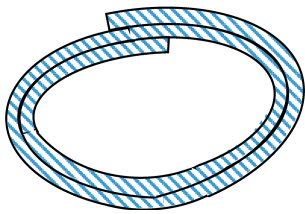


Figure 8.3: Relationship between the radius of curvature and n values for laminates $[0/90_n]$.

8.2 Variation in mechanical properties

The A-cones, as presented in Chapter 7, have some degree of overlap. For cones such as $[C_1/R_2]$, there are two layers along the radial direction and one layer along the circumferential direction. It has a thickness of three layers (0.375 mm). The buckling strength of this A-cone was determined to be about 16,000 N. A composite cone made by filament winding has a thickness of about 2.3 mm and a measured buckling load of 230,000 N [2]. Based on the unit weight, the A-cone and filament-wound cone have similar performance. It would be of interest to see how an A-cone that can support a load of 230,000 N can be made. The ratio between the two loads is $230,000 \text{ N}/16,000 \text{ N} = 14.375$. Knowing that the buckling strength is theoretically proportional to the cubic power of the thickness, this requires a thickness increase of 2.45 times.

One way to increase the wall thickness of the A-cone (while maintaining the same curvature) would be to have a few layers of overlap. Figure 8.4 shows the situation of two layers of overlap. Two layers of overlap would increase the buckling load by 8 times (i.e., 128,000 N). Three layers of overlap would theoretically increase the buckling load by 27 times (i.e., 432,000 N).



A-cylinder or a cone with two-times overlap

Figure 8.4: Cross section of an A-cylinder or A-cone with one complete layer of overlap.

The procedure to bond the overlapping layers together would be as follows:

- Apply epoxy resin to the relevant surfaces.
- Place the lay-up inside the cavity of an outer mold with a diameter slightly larger than the diameter of the desired A-cone.
- Insert a silicone bag inside the lay-up.
- Pressurize to inflate the silicone bag.
- Heat to cure the resin.

Figure 8.5 shows a possible setup for the bonding.

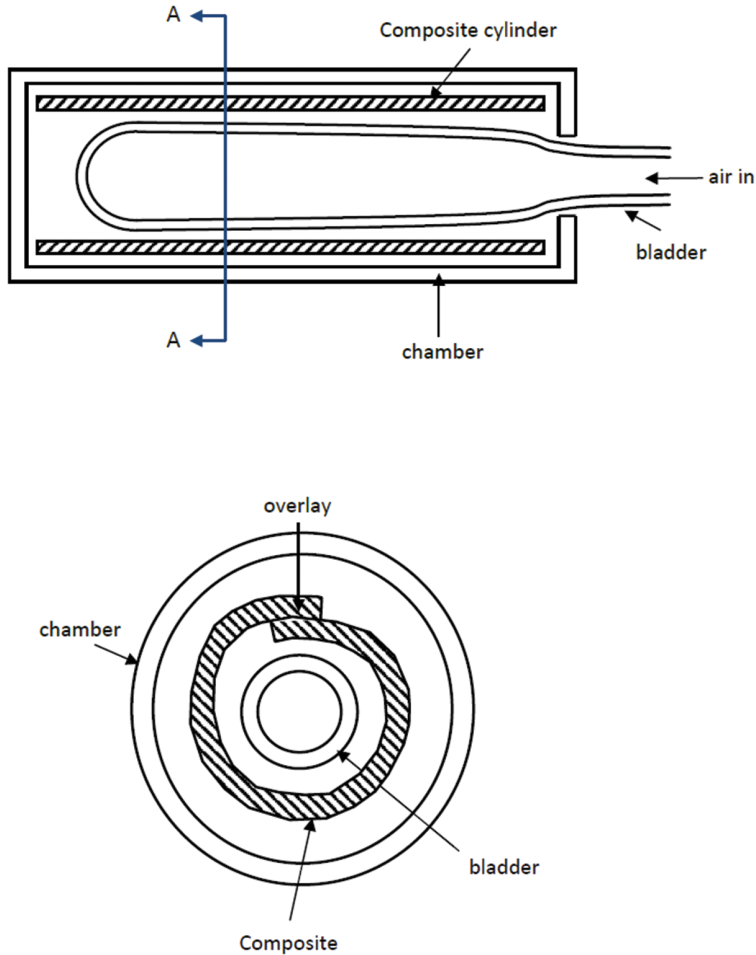


Figure 8.5: Possible setup for the bonding of the overlaps.

8.3 A-cylinder with an elliptical cross section

Chapter 7 shows the procedure to make A-structures such as A-cylinders or A-cones. These have cross sections in the shape of circles. For A-cylinders having a cross section in the shape of an ellipse, a similar procedure can be used with a modified lay-up sequence. Figure 8.6 shows the lay-up configuration for A-cylinders with an elliptical cross section. The final configuration, after curing and cooling to room temperature, is shown in Figure 8.7. In Figure 8.6, the top figure (a) shows the elliptical cross section with two regions. Region A represents the flat region of the ellipse (the region with flatter curvature), and region B represents the curved region of the ellipse (the region with sharper

curvature). In Figure 8.6b, layer 1 is made of fibers along the length of the blank (call it the x direction), and layer 2 consists of fibers along the y direction. In Figure 8.6c, the third layer consists of partially filled segments with fibers along the x direction. In this layer, segments with filled fibers represent region A, and segments with no fibers represent region B. This is because in region A, the whole laminate (consisting of three layers) is symmetric and as such will remain flat after curing and cooling to room temperature. On the other hand, region B of the laminate has only two layers and is an unsymmetric

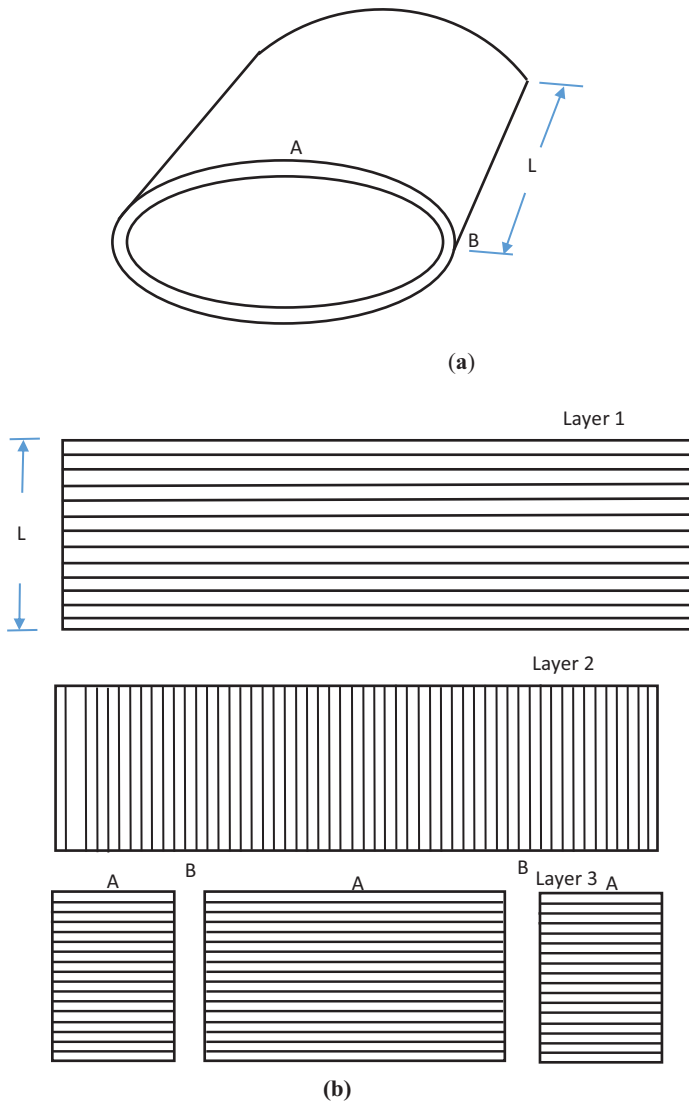


Figure 8.6: Lay-ups for cylinders with elliptical cross-sections.

laminate. After curing and cooling to room temperature, region B will curl up while region A may not. The overlapping will occur within region A. After secondary bonding, the configuration shown in Figure 8.7 may be obtained.

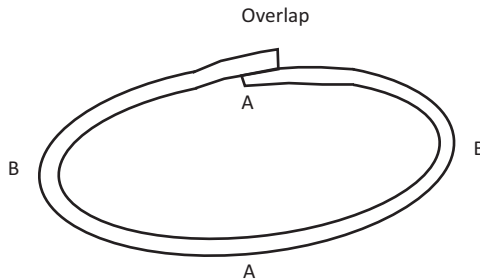


Figure 8.7: Final configuration of an A-cylinder with an elliptical cross section after curing and cooling to room temperature.

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